FFT APPLICATIONS TO ACOUSTIC PROPAGATION AND SONAR BEAM FORMING SIMULATION PROBLEMS

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I. INTRODUCTION

The development and subsequent application of the Fast Fourier Transform (FFT) is a matter of such extensive record that it need scarcely be mentioned here. The FFT has revolutionized many aspects of computational mathematics, not the least of which is the field of digital signal processing. Indeed, progress in this area has been of such magnitude as to diminish the significance of the FFT as a computational tool in other areas; many users have, in fact, come to equate the term "FFT" with a *time-to-frequency transform* and "IFFT" (Inverse FFT) with a frequency-to-time transform. That this is only one of many potential uses has been pointed out[1] in the past; we will here discuss two which have relevance to the Navy because of their possible application to advanced ASW training devices.

II. DISCUSSION

It has often been shown [2,3] that the finite, discrete Fourier Transform (DFT) given by

$$A_j = \frac{1}{N}\sum_{k=0}^{N-1} X_k\, e^{-2\pi ijk/N} \qquad j = 0,1,2 \ldots N-1 \tag{1}$$

can be treated in its own right as a valid orthogonal transformation which takes the sequence X_k into the sequence A_j. The properties of the DFT which may then be derived, while paralleling those of the Fourier Integral Transform, are nevertheless *exact properties and not merely approximations to the properties of the* integral transform. It nonetheless can be quite convenient to regard the DFT as an approximation to the Fourier Transform

$$A(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} X(t)e^{-i\omega t}\,dt \tag{2}$$

when the error involved in truncating the range of integration can be neglected. A wide variety of important integrals in the fields of mathematics, physics, and engineering may be put into the form of (2) or the inverse transform given by

$$X(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(\omega)e^{i\omega t}\,d\omega \tag{3}$$

and thus can be suited for evaluation via the DFT, and hence the FFT, which is simply an efficient algorithm for the computation of (1).

Within the field of ASW operations, and consequently ASW training devices, there are two areas of primary importance whose theoretical treatment involves integrals of the form of (3); that is, underwater acoustic propagation and sonar transducer array analysis. A realistic simulation of the effects due to diverse phenomena in each of these areas is essential if the ASW trainers themselves are to be realistic, yet it is precisely these effects for which it has been most difficult to generate accurate real-time models. There are varied reasons for this as discussed in the following paragraphs.

In the case of ocean modeling, one problem is that even the most advanced theoretical formulations designed to execute on large computer systems involve considerable simplification in the original formulation of the problem, the mathematical solution of this problem, or both. Tolstoy and Clay [(4)] discuss this state of affairs in considerable detail; we may summarize generally by saying that, except in shallow water and at short ranges, the most we can do is to predict the general features of the acoustic field. Detailed knowledge of its fine structure as a function of a multitude of environmental parameters is beyond us. If this is true of even the best work existing at this time, it is even more so with models which must process within the real-time constraints of a trainer. Such ocean models have typically taken the form of simplified versions of some more advanced model, mass storage of the output of more complex models which is then assessed on a table look-up and interpolation scheme, or some combination of the above.

In the case of sonar transducer array analysis and beam forming, we have a different problem. The theory, to a large degree, of such multiple arrays is well understood. If computer time and storage permit, an excellent model of the array beam-pattern may be generated via the DFT. For simulation purposes, though, the prohibitive computation times involved have necessarily resulted in complex, expensive, and somewhat unreliable hardware methods* for simulating the effects due to sonar transducer arrays.

The application of the FFT can change this situation. To see how this is so, we will turn briefly to the theory, beginning with the case for underwater acoustic propagation.

With the assumption of a sound velocity profile that is a function of ocean depth only, with plane, parallel boundaries at surface and bottom (see Figure 1), we may write [4,5]

$$\Psi(r,z) = \int_{-\infty}^{+\infty} g(z,\ z_0,\ \xi)\ H_0^{(1)}(\xi r)\xi d\xi \tag{4}$$

where $r = (x^2 + y^2)^{\frac{1}{2}}$ $\qquad \xi$ = horizontal component of $\vec{k}$

z = receiver depth $\qquad \vec{k} = \dfrac{2\pi f}{c(x,y,z)}$

z_0 = source depth

*For example, a common hardware method uses tapped delay lines and a commutator. In some cases inverse beam formers are used whose output is what the compensators after the hydrophones normally receive from the hydrophones. The taps on the delay lines are selected by an electromechanically driven commutator. Hence, the reliability problem with the mechanical contacts.

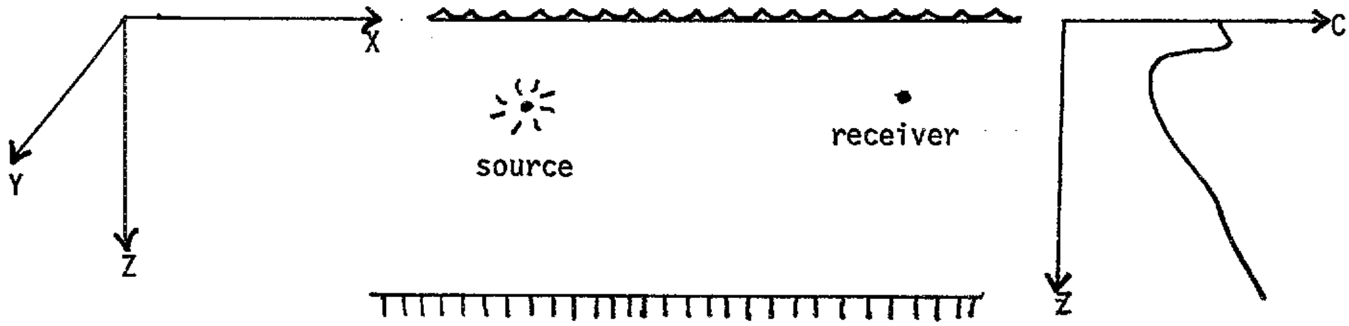

FIGURE 1 COORDINATE SYSTEM, OCEAN GEOMETRY, AND TYPICAL OCEAN SOUND VELOCITY PROFILE

as the solution to the time-independent acoustic wave equation for a point source at $\vec{r}_0$,

$$\nabla^2\Psi(\vec{r}) + k^2(\vec{r})\ \Psi(\vec{r}) = \delta(r-r_0) \tag{5}$$

where

$$\vec{r} = (x,y,z)$$

Marsh and Elam [6] were the first to note that when the Hankel function of zero-th order, first kind is approximated by the first term in its asymptotic expansion.

$$H_0^{(1)}\ (\xi r)\ \cong \left(\frac{2}{\pi i}\right)^{\frac{1}{2}} (\xi r)^{-\frac{1}{2}}\ e^{i\xi r} \tag{6}$$

the preceding acoustic field integral, Equation (4), may be put into the form of the Inverse Fourier Integral Transform (3) and hence may be evaluated via the Fast Fourier Transform. F. R. DiNapoli of NLONLAB NUSC, has greatly extended their work [7,8,9,10] and the result is a mathematical treatment of underwater acoustic propagation phenomena which is as fundamentally rigorous as any model available, yet which offers the potential of extremely efficient machine usage due to the speed of the FFT.

The application of the FFT to sonar array analysis and beamforming simulation is more straightforward, as it is well known [11,12] that under conditions where the source is sufficiently far from the receiving array (see Figure 2) to assume Fraunhoffer conditions, the received amplitude at the array will vary with angle

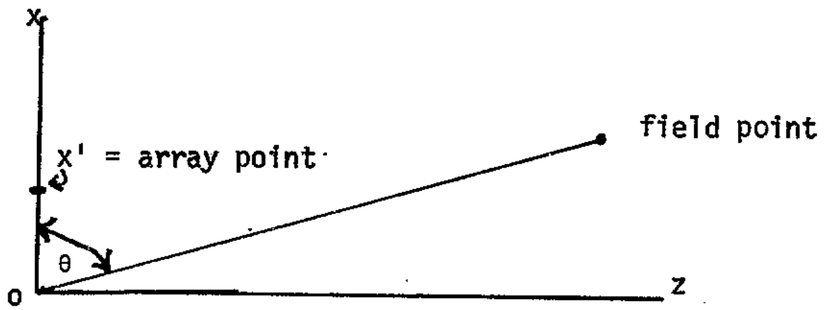

FIGURE 2 PROBLEM GEOMETRY

as

$$p(s) = \int_{-\infty}^{+\infty} q(u)e^{2\pi isu}\,du \tag{7}$$

where

$s = \sin\theta$ $\qquad$ $q(u) = \lambda f(x')$

$u = x'/\lambda$ $\qquad$ $f(x')$ = source distribution function

x' = source coordinate

This is, of course, the one-dimensional case. Usage of this form is acceptable for either one-dimensional arrays or in cases where the geometry of the array is such that the source distribution function may be represented by a form such as

$$F(x',y') = f(x')\,g(y')$$

which is equivalent to saying that we may separate variables. More recalcitrant cases may be handled via multidimensional transforms, though these are quite unwieldy. Equation (7) has the inverse

$$q(u) = \int_{-\infty}^{+\infty} p(s)e^{-2\pi isu}\,ds \tag{8}$$

which allows us to specify the amplitude and phase of the received signal, knowing the radiation pattern. It is immediately apparent that (7) and (8) are simply a Fourier transform pair and are susceptible to FFT evaluation. In fact, as C. M. Rader notes[13], one of the earliest and clearest FFT applications was in the field of radar antenna array signal evaluation (i.e., beamforming), on exactly equivalent problems. Extensive work has been done in the area of beamforming, i.e., processing the received signal. The innovation here is in the application of the FFT to the array pattern, with the purpose of synthesizing the output at the array transducers.

It would be incorrect to say that this work is all directly applicable to trainer use. As with many other desirable mathematical formulations, extensive work is required to render the models both small enough and rapid enough for computer processing in real time. For example, the NUSC ocean model which we have been discussing, the Fast Field Program (FFP), uses 100,000 words of memory on a large machine, in FORTRAN. An FFP based on the original NUSC work has been developed by Singer-Simulation Products Division R&D investigators which will process the same cases using less than 16,000 words of memory on a typical 16-bit minicomputer, in FORTRAN. This was achieved in the course of an internal research and development program during which a new method of computing certain key portions of the FFP program was found. That is, the input to the FFT in this program is obtained from a sequence of points which may be computed through the use of Bessel functions of varying order and fixed argument. In some cases [14] these functions can be unstable when calculated on a computer; complicated expansions are needed in such cases to obtain valid results. During our work with the FFP, a method due to John G. Wills [15] was discovered which avoided this problem. A modification of Wills' method enabled us to completely rewrite the program so as to obtain more efficient execution.

Although the two programs require roughly the same execution time on a given machine, about 90 seconds on a CDC-6600, it is estimated that when programmed in assembly language for a 16-bit minicomputer with floating-point arithmetic hardware, execution times on the order of 30-45 seconds can be achieved. Future mathematical simplifications, optimization and restructuring, or use of the Singer-FFP with an efficient off-line storage system promise to yield a fundamentally valid model capable of meeting the timing constraints of a real-time training device.

The development of a software array simulation model requires care in the application of the DFT (Equation 1) to Equations 7 and 8. Consider, for example, the generation of the pattern from the transducer array, in this case a one-dimensional array. We have

$$p(s) = \int_{-\infty}^{+\infty} q(u)e^{2\pi isu}\,du$$

To evaluate this integral via the DFT, let u and s be approximated by

$$s_n = s_o + n\Delta s$$

$$u_m = u_o + m\Delta u$$

and

$$\Delta s\Delta u = \frac{1}{N}$$

then

$$p_n(s) = \sum_{m=0}^{N-1} q_m(u)e^{2\pi i(s_o u_o + s_o m\Delta u + u_o n\Delta s + mn\Delta u\Delta s)}$$

and

$$p_n(s) = e^{2\pi i u_o s_n} \sum_{m=0}^{N-1} G_m(u)\, e^{2\pi imn/N}$$

where

$$G_m = e^{2\pi i s_o m\Delta u} q_m(u)$$

Thus, $p_n(s)$ is given by the DFT of G_m, and as such is suitable for application of the FFT.

Valid application of the discrete transform is highly dependent on the determination of suitable sampling criteria. Too large a transform will render the method unwieldly, while too small a transform will result in aliasing within the FFT. Further, the sampling intervals in the two domains are not independent and trade-offs are necessary to obtain certain required angular resolutions while still correctly sampling the transducer array. To illustrate, suppose we consider an eleven element line array with inter-element spacing of $\lambda/2$. The total range to be sampled in the dimensionless u-domain is, therefore, 5 units. Maximizing the use of a given number of sample points would require

$$\Delta u = \frac{5}{N}$$

which would, in turn, result in

$$\Delta s = \frac{1}{5}$$

and thus between 0^{0} and 90^{0} only five points would be obtained; a clearly unacceptable result. Further, this result is independent of the number of points used in the transform. Picking an angular resolution on the order needed merely makes Δu so large that most elements are missed completely. The only alternative is to allow the size of the sampled u-domain to be arbitrary; that is

$$x'_{max} = k\lambda$$

$$u'_{max} = \frac{x'_{max}}{\lambda} = k$$

and therefore

$$\Delta u = \frac{k}{N}$$

$$\Delta s = \frac{1}{k}$$

necessarily. Then Δs is chosen as desired, say (1/128) for acceptable resolution in the 0^{0} - 90^{0} range. Then it is required that

$$k = 128$$

and, in general

$$k = 1/\Delta s$$

The specific case we have been discussing, the 11-element line array, occupies a total distance of 5λ, while

$$x'_{max} = 128\lambda$$

Thus, in the dimensionless u-domain, the line array consists of only about 4% of the sampled region, clearly a waste of computer time and space. This whole problem of trade-offs in the required transform size versus desired resolution in the output is the exact analogue of the more usual problem in spectral analysis of choosing a transform size and time window so as to obtain both correct sampling in the time domain and adequate resolution in the frequency domain.

Work with large transducer arrays or two- and three-dimensional arrays has not as yet been attempted. It must be noted that, since multidimensional transforms require extensive amounts of computer time, it will probably be true that to correctly simulate an array it will be necessary that the array and its pattern possess sufficient symmetry that a one-dimensional transform is suitable for modeling it. For example, a planar array can, using the separation of variables technique in Cartesian coordinates, be handled with a one-dimensional transform. Similarly, a spherical array in polar coordinates can be so handled.

The preceeding summarizes our work in attempting to develop more advanced simulation models for ocean acoustic propagation and sonar beamforming. Discoursing for a moment on a more advanced plane it becomes interesting to note that a possibility exists for relating these two models; that is, tying the beamforming simulation directly into the ocean model. This can be seen by returning to Equation (4)

$$\Psi(r,z) = \int_{-\infty}^{+\infty} g(z, z_o, \xi) \; H_o^{(1)} (\xi,r)\xi d\xi$$

where it has been noted that ξ is the horizontal component of the wave number in the separated-space form of the wave equation. That is, when

$$c(\vec{r}) \equiv c(z)$$

Equation (5) may be separated into two equations, one giving the horizontal dependence of the wave on position and the second,

$$\frac{d^2Z}{d\,z^2} + \left[k^2(z) - \xi^2\right] Z = \delta(z-z_o) \tag{9}$$

giving the wave function Z(z) in the vertical direction. Here

$$k(z) = \frac{\omega}{c(z)} = \frac{2\pi f}{c(z)}$$

and

$$\xi = k(\vec{r}) \sin\theta$$

in Figure 3.

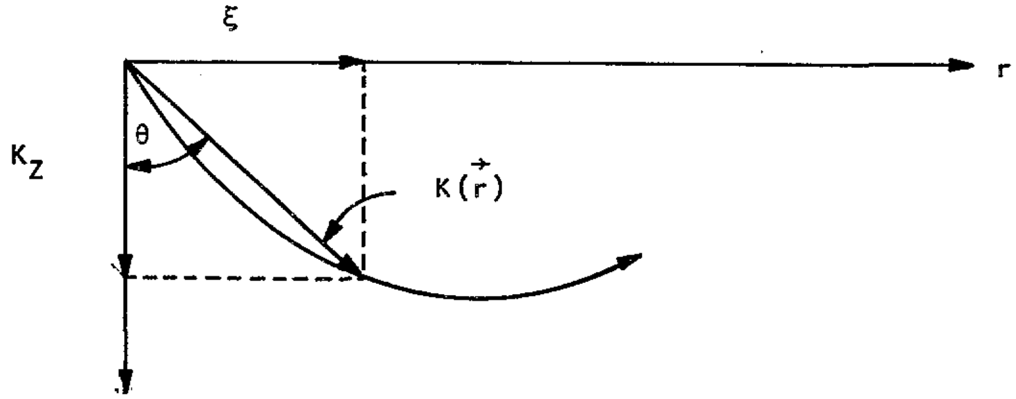

FIGURE 3 SAMPLE RAY WITH COORDINATE SYSTEM

Here, to use an analogy from ray theory, the separation constant obviously corresponds to the horizontal component of the wave number vector $k(\vec{r})$ at some point on a given ray. The integration in (4) corresponds to summing the contributions to the field over all possible ξ and thus over all possible angles at which contributory rays may exist. If, then, it may be determined that a given array should boost the received field at some angle, then the value of the integral which is applied as input to the FFT can be boosted at the appropriate angle. We must hasten to note at this point that the actual case is not quite this simple. The problem is that wave and ray optics do not correspond as exactly as we have been indicating here, and the acoustic field integral is a wave optics formulation. Work is currently being conducted at NUSC in this area.[16] It remains at this point sufficient to say that the possibilities are interesting.

Along these same lines, another possibility for further research also exists in the beamforming area. Advanced sonar signal processing systems are showing that it is much easier to process signals in beam space. These advanced techniques require closed loop analysis in space and time for optimum beamforming. In beam space the beams are orthogonal and hence the equations become decoupled. This considerably decreases the number of calculations necessary. For these systems the beam will be optimized for the target with associated background noise and interference. Therefore, the simulation will just have to enter degradation from the ideal to account for errors in the simulated system. The entire beamforming could then be tied directly to the ocean model, so that the input would be to the ocean model and the output would be the result after beamforming.

III. CONCLUSIONS

The FFT has been discussed in terms of its potential uses in two fields of importance in ASW simulation and training; e.g., the direct evaluation of the acoustic field integral for underwater propagation and the determination of the output of a sonar hydrophone array from its beam pattern. The problems encountered in adapting work in these fields to the requirements of simulation are discussed as well as some of the solutions which have been obtained. Possible directions for advanced work have been indicated.

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