THE SPECTRUM OF MULTIPLE-SAMPLED NON-CAUSALLY INTERPOLATED WAVEFORMS

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INTRODUCTION

Recent advances in computational and storage hardware have made it possible to produce complex signals in real time to simulate the diverse sounds that are picked up by hydrophones in sonar systems. The generation of such sound signals are important to shorebased simulation trainers that use operational equipment in-board from the hydrophone and its preamplifiers. This type of trainer design is known as a "stimulator", since the output signal from the simulator "stimulates" or drives operational equipment. In contrast to this class of trainer design is the pure simulator, where everything in the real world is modeled, and the output drives meters, cathode ray tubes (CRT) and earphones to simulate the instrumentation, display and sounds a sonarman sees and hears in an operational sonar room.

The approach used in the design of a sonar trainer depends heavily on the long term objectives of a program, and the cost effectiveness of the approach in meeting those goals. It is obvious that a mix of methods is possible. Given that a hybrid approach satisfies the needs of the customer, the degree of mix depends on the most technically expedient locations into which simulated signals may be injected into the operational equipment and still provide the most cost-effective trainer. Without delving into the pros and cons of simulation versus stimulation, this paper deals with the spectrum of sound generated by some new digital synthesis methods. The need to study the spectral characteristics of the synthetic underwater sound is fundamental to the requirement of providing a realistic simulation that allows effective training.

The need for new techniques in trainer design can be traced to the increased sophistication of new sonar systems. Some factors that affect the specification of sonar trainers are discussed in the next two sections. New trainer requirements caused by sonar technology advancements are discussed through a synopsis of sonars. The complexity and diversity of sounds encountered by sonars and what this means to synthetic sound generation is discussed later. Given that a digital sound generator is used, the spectra of different generation schemes are discussed. The last section addresses the pros and cons of digital sound processing, and the technical implications of conducting all of the processing in the digital domain for future sonar stimulation trainers.

SONARS AND TRAINERS

SONARS

The most basic function of sonars is to couple and convert sound energies from the ocean to some form that is useful to a human operator. In its simplest form, a sonar consists of a hydrophone and amplifiers that drive a set of earphones or speakers. The hydrophone couples and converts underwater sound energies to electrical signals that are amplified to drive earphones or speakers. To improve the directional discrimination of the sensors, several hydrophones are used to simultaneously pick up signals from the environment at physically separate locations. These signals are then electronically processed to reinforce signals from a given direction and to discriminate against signals from all other directions, otherwise known as beamforming. The resulting information is either presented as an audible sound through earphones or displayed as waveforms on a CRT display.

One main job of a sonar operator is to identify objects through their signal signatures. To help him in this task, sophisticated electronic signal analysis equipment has been introduced into modern sonar systems. One such device is the realtime spectrum analyzer that displays the spectra of the input signals to help the sonarman identify targets through spectral line families. It is, therefore, important that a stimulator used for training purposes be capable of providing similar spectral characteristics on the analysis equipment.

Sonars are either active or passive. Both types of sonar use hydrophones to pick up sounds from the ocean environment. Active sonars have the added ability to launch a self-initiated sound pulse to produce echoes from objects in the environment. Active sonars can also be used in a passive mode when their hydrophones are used to pick up sounds already present in the environment. Since the problem addressed in this paper deals with simulating the driving signals to a sonar system in a trainer, the effects of active sonar pings are not discussed.

SONAR TRAINERS

Devices designed as aids in the training of student sonarmen range from complicated team trainers to single station devices. Both extremes are designed to simulate the "real world" to the extent that meaningful training can be accomplished without placing the student in an actual operational situation. Such shorebased training has been shown to be very cost effective. Furthermore, simulators allow the instructor the freedom to control the "physical" environment in such a way that is not practical to provide by any other method.

For the sake of completeness, the basic methodology used in the design of sonar trainers is outlined with reference to Figure 1, which shows the principal components of a trainer. All sound sources are mathematically modeled to provide proper signatures. These sources are broadly divided into man-made sound sources and natural sound sources. Target signatures are assumed to consist of all man-made noises, due primarily to shipping activities. Natural signatures encompass environmental conditions, such as sea state and rain squalls, and biological sounds emitted by such ocean inhabitants as whales and porpoises. The transmission characteristics of the ocean are modeled mathematically to provide the proper dispersion, attenuation and distortion to the sounds generated by the various sources. At this juncture, the question of whether simulation or stimulation techniques are to be used in a particular device must be decided. If simulation methods are chosen, then the effects of the sonar hardware must be modeled to produce the proper output displays. These effects include beamforming, spectrum analysis, and any other signal processing that might be performed by the operational system. The resulting system must also respond properly to any knob adjustments that might be available to the student. On the other hand, if the device is to stimulate operational equipment, electrical signals must be generated with proper characteristics to produce the desired responses in the sonar system. Proper relative phase and delays are necessary, primarily, because of the way in which they are used to generate beams in hydrophone arrays. An elementary discussion on phased arrays is given in the next section to illustrate this point.

SONAR ARRAYS

A sonar array is a somewhat ordered distribution of hydrophones in space. Two examples of actual arrays are depicted in Figure 2. In one case, the array consists of a network of hydrophones hanging from buoys dropped from helicopters to form a preassigned pattern. The second example is an array of hydrophones fabricated in the form of a cylinder. In each case the hydrophones are spatially fixed, and any sound impinging on the set of hydrophones will produce signals that are phase related. These signals are then electronically processed to produce a narrow beam that provides a directional sensitivity. Let S_1, S_2, ... S_n be signals transmitted by hydrophones

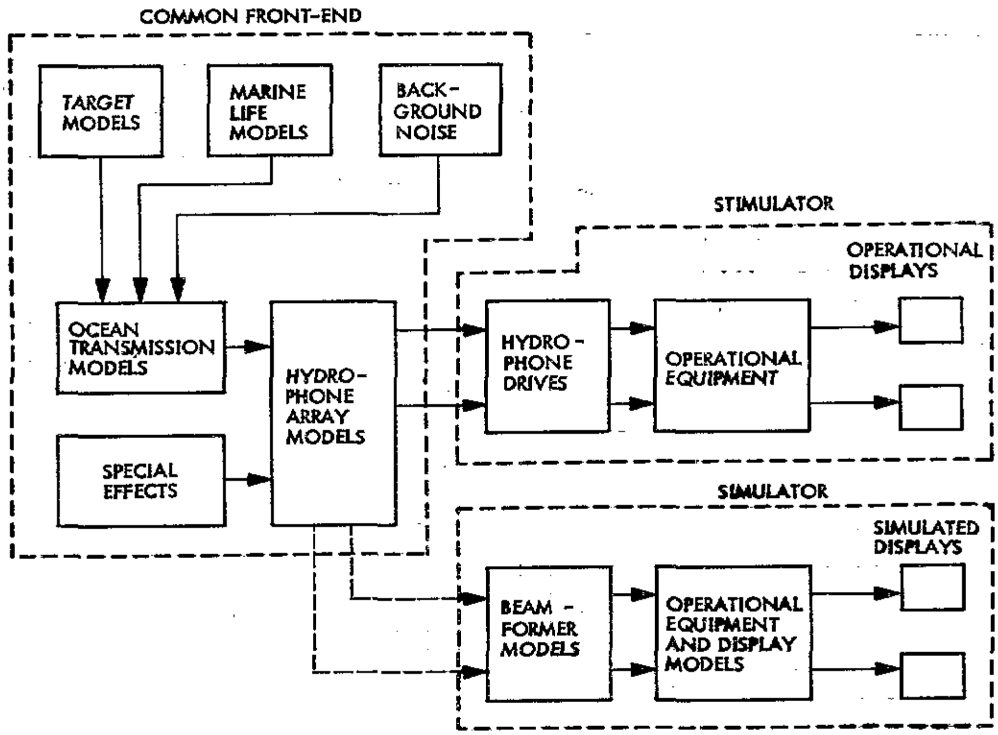


Figure 1. Sonar Trainer Functional Block Diagram

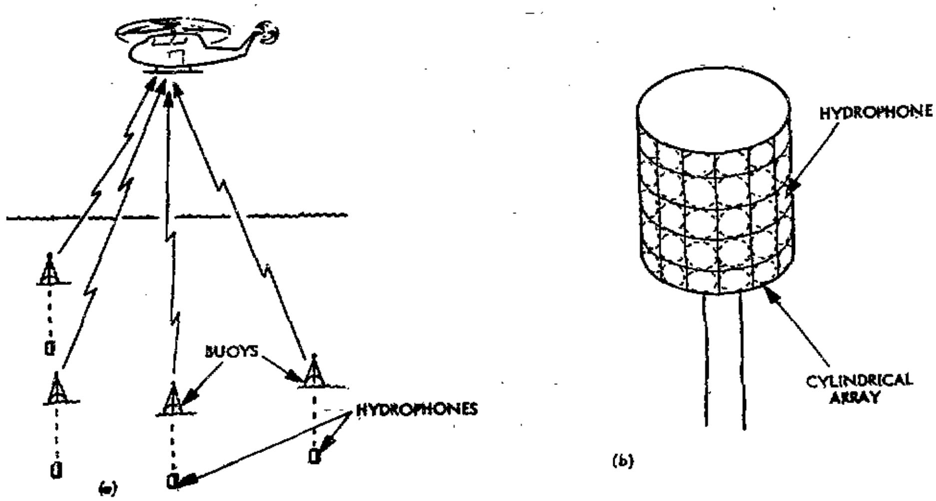


Figure 2. Hydrophone Arrays

$h_1, h_2, \ldots, h_n$, respectively, to a processor that individually delays each signal and then forms a weighted sum of the resulting signals,

$$S(t) = \sum_{i=1}^{n} a_i S_i(t - \delta_i) \quad (1)$$

where the a_i's and δ_i's are the weighting functions and delays placed on the signals from the corresponding hydrophones, h_i.

By a judicious choice of delays, it is possible to reinforce signals arriving from a given direction relative to the hydrophone array, and accentuate the mutual interference of signals arriving from other directions. A proper choice of weights will intensify the process of selective reinforcement and interference. Such a process is shown in Figure 3. Note that by changing the delays and weighting on the hydrophone signals through switches, it is possible to change the beam characteristics without altering the hydrophone array itself. The important fact is that signal phase plays a key role in sonar signal processing.

TRAINER FIDELITY

A problem trainer manufacturers constantly battle with is the level of realism needed for training versus the cost of providing it. Typically, greater realism means models that are more complete, and hence more complex. This greater model complexity usually implies more processing which, in turn, translates to greater cost. Sonar trainers are not void of such considerations.

In the past, sonar trainers had to produce signals that were subjectively judged to be "correct" by experienced sonarmen. This involves providing signals with the right mix of background or ambient noise, target signatures, and directionality effects to satisfy the expert listener. With the addition of signal processing devices and improved hydrophone designs, simulated hydrophone signals used to stimulate operational sonar equipment provide signals that also have the correct spectral and waveform characteristics. Since all of these features require more hardware and software, the question becomes one of choosing those factors that the higher resolution systems pick up that are important to effective training. This problem is not discussed any further, but is used as a backdrop to the techniques analyzed later to provide the higher resolution without a commensurate increase in hardware complexity.

SYNTHETIC SOUND GENERATION

DIVERSITY OF SOUNDS

Sounds that form the composite signal picked up by a hydrophone can be broadly classified into three groups, those noises caused by water movement, by marine life, and by man-made sources. To provide an appreciation for the diversity encountered, each class of noisemaker will be discussed briefly.

Noise generated by water movement has been measured by many research teams under a variety of conditions at numerous locations. Figure 4 shows an average spectra of water-movement-related underwater sound, as a function of sea state, which is commonly measured in terms of wave height. It is interesting to note that the spectrum is not flat, but falls with rising frequency. The minimum water noise can be attributed to inherent thermal noise in the water and the motion of general marine life present in the environment. This minimum is rarely exceeded in actual measurements. The origin of noise due to water movement ranges from water splashing on foreign objects, such as a shoreline and the hull of a ship, to the impact of water on water as in breaking of wave crests. Other examples of noise due to water movement can be found in the literature on sonars [1].

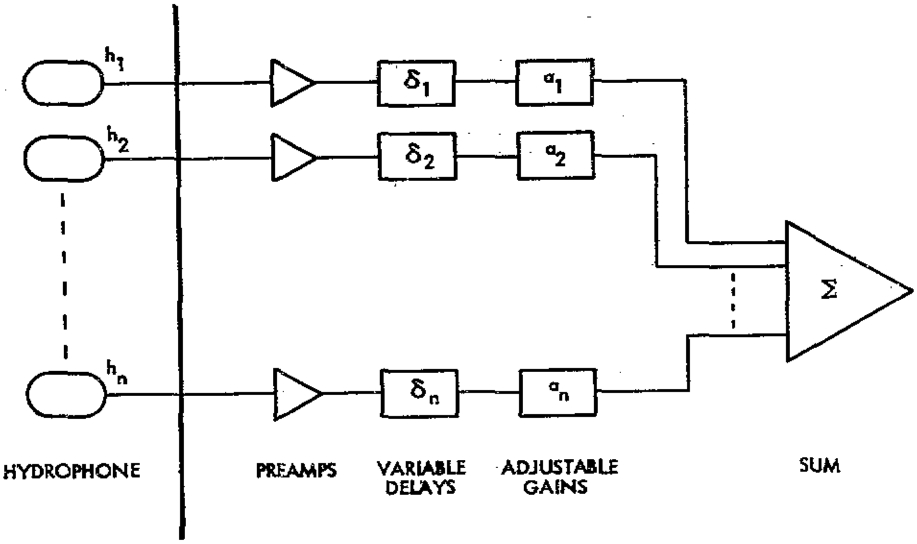


Figure 3. **Beamforming**

Marine life of all types also contributes to the general noises found in the ocean. Many marine creatures emit sounds that have characteristic signatures. The toad fish is one which emits a loud sound, much like a violin string stroked by a bow. Others, like the porpoise, produce such a variety of sounds that it is useless to try to identify them. For instance, the porpoise is known to bark like a dog, gobble like a turkey and sometimes emit a distinctive bubbling whistle. There are other marine inhabitants that produce practically imperceptible sounds that become annoyingly audible only when huge numbers of them are active at the same time. Two such creatures are the snapping shrimp and a small fish called the croaker. Noise generated by this last group of noisemakers is both seasonal and locality-bound.

The principal source of man-made noise in the open seas is shipping. Sounds from a given ship depend on its type, size, design, and mode of operation. The sound actually received by a hydrophone depends on the depth and temperature profile of the ocean, as well as the relative location of the ship emitting the sounds to the hydrophone picking them up. However, the sounds emitted by ships are very distinctive, and are effectively used to identify the source. A trainer, therefore, must duplicate shipping sounds to such a degree of exactness that a student will be able to transfer the knowledge he gains from the trainer to the operational world. This usually means a reasonably faithful reproduction of the sound waveform and spectra.

The noise spectrum level of shipping does not differ much from general water noise, with the exception of a steeper negative slope and the manifestation of distinct peaks. Engine and propeller noises produce characteristic frequencies, with the latter being complicated by such factors as cavitation [2], mechanical vibration and singing [1], all resulting in a complex modulation of the basic noise.

The purpose of this discussion is to point out the diversity of noise sources, types and effects that a trainer must produce to provide a "realistic" signal to operational sonar systems. The very breadth of characteristics require a parametric sound source that will produce a proper composite signal to each hydrophone feed-in line.

DIGITAL SOUND GENERATION

In the frequency domain, the noise input to the hydrophones is seen to consist of pink* noise, with random line frequencies occurring either singly or in families. These signals are reflected in the time-domain as a complex composite of noise, single frequency signals and multiple frequency signals. As found under normal operating conditions, the signal characteristics are also time variable.

The composite sound generator in a sonar stimulator is typically a self-sustaining parametric system that changes sound characteristics only in response to environmental changes. A given

*Pink or colored noise is used to describe noise with a non-uniform spectrum in contrast to white noise which is used to describe a uniform spectrum.

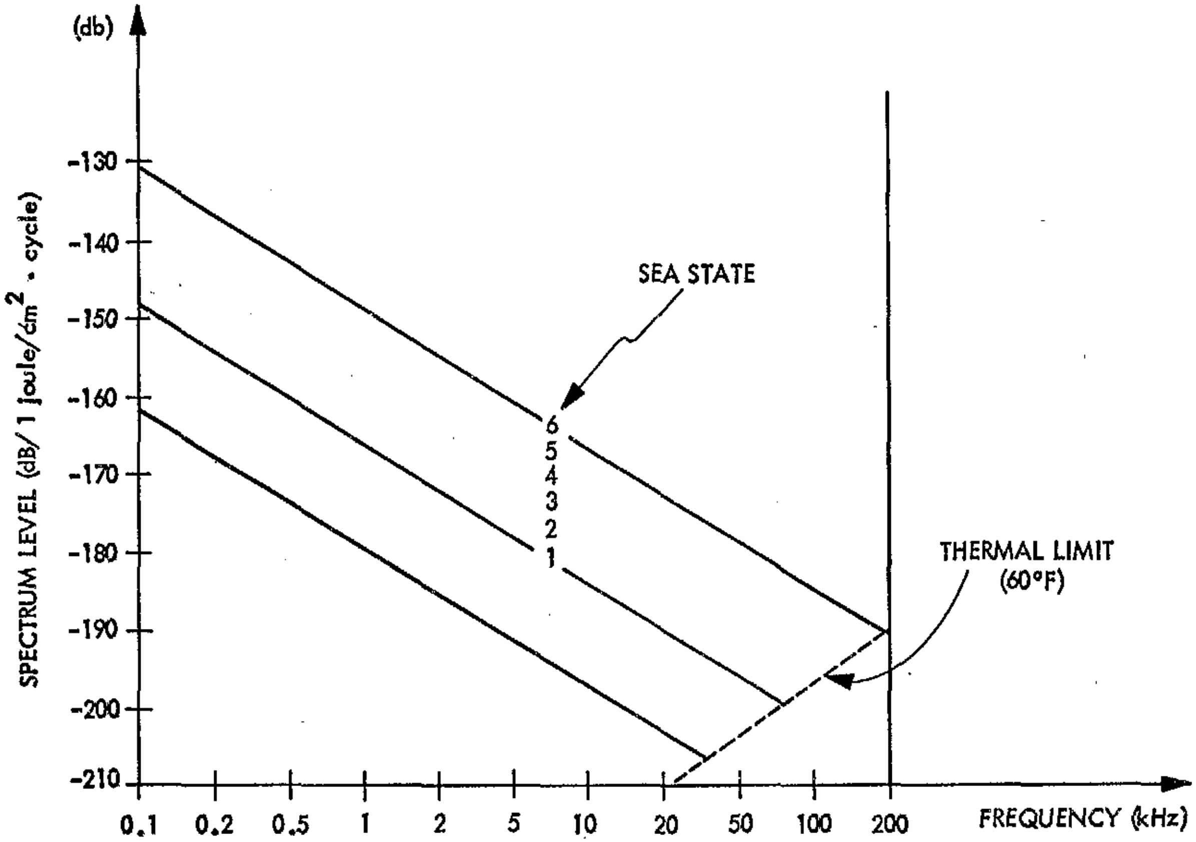


Figure 4. Amplitude Spectrum of Water Noise

set of characteristics is usually determined through software models calculated by a general purpose computer. The resulting parameters are then input to the sonar sound generator at prescribed intervals to produce an essentially time-varying scenario.

Since the sound generator is self-sustaining between updates, much of the data needed to produce the proper sound signals must be stored. To conserve hardware, the data is stored in as coarse a resolution as the simulation will allow without compromising the nature of the signal used to stimulate the sonar system. This discussion has assumed the synthetic sonar sound signal to be digitally generated. A digitally generated sound signal is represented by a series of binary numbers generated in the following fashion. A continuous wave is periodically sampled, and the resulting discrete amplitudes are then quantized and represented by a binary number. This is depicted in Figure 5.

From sampling theory it is known that if the original waveform contains no signals having frequencies greater than ω_c, then by sampling the analog signal at a frequency ω_s which is greater than $2\omega_c$, the original signal can be recovered completely when low-passed by an ideal filter with an upper cut-off frequency of ω_c. In practical systems, the ideal conditions of sampling theory are not achievable, therefore higher sampling rates and different interpolation schemes must be used. The interpolation process is typically performed by a digital-to-analog (D/A) converter. The output is then filtered by a realizable low-pass filter to discriminate against frequency components introduced by the digitizing, quantizing, and interpolation processes.

When digital data representing various waveforms sampled at different rates are combined to form a composite waveform, incorrect values will result everywhere except where the samples of all component waveforms coincide. One solution to the problem is to convert each signal to its analog equivalent and perform all the combining processes either directly in the analog domain, or in the digital domain after synchronizing the data through a common redigitization. An alternate method is to estimate the values of all the waveforms at the desired sampling time by interpolating between the bracketing samples and using the estimated values in the processing. The latter method has the advantage of keeping all the processing in the digital domain without the undesirable steps of converting and reconverting between the digital and the analog domain. The net effect of estimating the values of a given waveform between known samples is a multiple sampling of the original analog signal. The "sampling" in the digital domain may be at a higher or lower rate than the original sample, and may even be a mere constant delay in the sampling function. Whatever the condition, the method of interpolation and re-sampling format affect both the amplitude and phase spectra of the original signal. Since amplitude and phase errors in the spectral domain effect the ultimate signal displayed by the sonar equipment, their effects must be carefully studied.

Several interpolation schemes have been studied to determine the spectral effects of each on the synthesized waveforms. Among the interpolation functions studied are polynomials, exponential functions and hyperbolic functions. Only the zeroth- and first-order polynomial

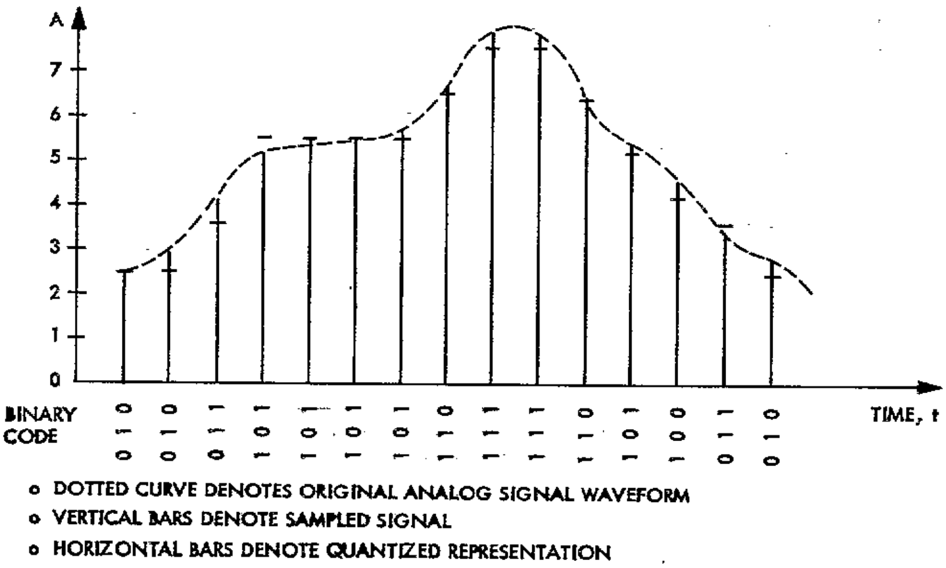


Figure 5. Digitized Signal and Corresponding Binary Code

interpolators and the effects of a delayed re-sampling function will be discussed in this paper.

THE MEANING OF NON-CAUSAL PROCESSING

Non-causality plays an important, though subtle, role in the processing used in simulation trainers. To appreciate its contribution, suppose that x(t) is the input to a system that produces an output y(t). By definition, the system is causal if y(t) appears only after x(t) is input to the system. In a trainer system where an overall system lag is permitted for processing, it is possible to compute new output signals from data that are previewed for processing purposes only. This property means that new data can be computed non-causally, thus allowing the interpolation algorithms of the previous section to mechanize without the knowledge of the operational sonar equipment being stimulated.

SPECTRUM OF DIGITAL SOUND

SPECTRUM OF SAMPLED SOUND

Suppose a spectrally shaped digital noise source is processed with other digital signal sources to produce the waveform shown in Figure 6a. Mathematically, the waveform in Figure 6a is an amplitude-modulated version of the pulse signal in Figure 6b, where the modulating signal is shown in Figure 6c. If the analog signal in Figure 6c is a true sonar signal, with the spectrum shown in Figure 6c', then the spectrum of the digitally generated sound is calculated by computing the convolution of the Fourier transform of Figure 6c' with the Fourier transform of Figure 6b', to give Figure 6a'. The derivation of these results are well known and can be found in any book on sampling theory [3]. The effects of sampling can be removed by filtering the waveform of Figure 6a with a low-pass filter that has a cut-off frequency of ω_c.

SPECTRUM OF INTERPOLATED SOUND

Starting with a sampled signal, it is possible to fill-in the intrasample region by some interpolating function. Two commonly used polynomial interpolators are the zero-order interpolator (sample-and-hold as in Figure 7a) and the first-order interpolator (linear interpolator as in Figure 7b).

The zero-order interpolator consists of a circuit that will hold an impulse input at an amplitude equal to the area of the impulse from the time it occurs, until some time $t = T$ later. At the end of this period T, the output is reset to zero in preparation for another impulse input. The interval T is usually the period of the sampling function p(t). Stated formally, the input of one impulse with area A occurring at time $t = 0$ is

$$e_{in}(t) \triangleq A\,\delta(t) \qquad (2)$$

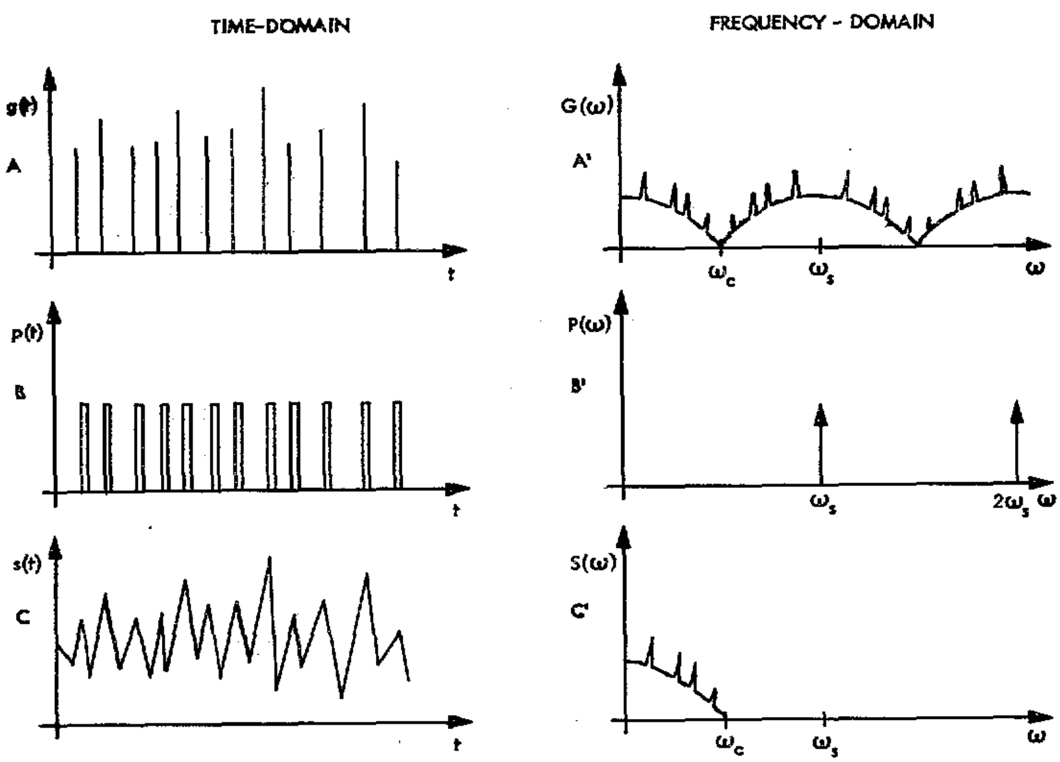
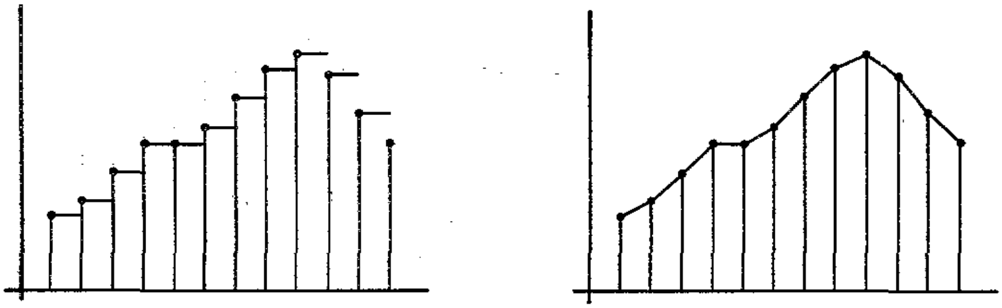


Figure 6. Spectrum of Sampled Sonar Sound



(a) ZERO-ORDER HOLD

(b) LINEAR INTERPOLATION

Figure 7. Zeroth- and First-Order Interpolation

The output response to $e_{in}(t)$ is then

$$e_o(t) = A\left\{u_{-1}(t) - u_{-1}(t-T)\right\} \tag{3}$$

where $u_{-1}(x)$ is defined to be a unit step occurring at $x=0$. The impulse response of a zero-order hold circuit is thus the difference between two unit steps shifted by T in time. Taking the Laplace transform of $e_o(t)$ gives the transfer function of the zero-order hold circuit to be

$$G_h(s) = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1-e^{-sT}}{s} \tag{4}$$

By substituting $j\omega$ for s and $\operatorname{Cos}\omega T - j\sin\omega T$ for e^{-sT} we get the amplitude and phase spectrum to be

$$G_h(\omega) = T\left|\frac{\sin\omega T/2}{\omega T/2}\right| \angle -90^\circ + \tan^{-1}\frac{\sin\omega T}{1-\cos\omega T} \tag{5}$$

The impulse function and the amplitude and phase spectrum of the zero-order hold circuit is given in Figure 8.

Suppose f(t) is a bandlimited function that is sampled by an impulse train p(t) to form $f^*(t)$. If the Fourier transform of f(t) is $F(\omega)$ and the period of p(t) is T, then the Fourier transform of the sampled wave $f^*(t)$ is given by

$$F^*(\omega) = \sum_{k=-\infty}^{\infty} F(\omega - 2\pi k/T) \tag{6}$$

If $f^*(t)$ is passed through the zero-order hold circuit, the resulting signal is given by the convolution of $f^*(t)$ with the impulse response of the zero-order hold circuit. That is,

$$\hat{f}(t) = f^*(t) * g_h(t) \tag{7}$$

where $*$ denotes the convolution operator. With $G_h(\omega)$ the Fourier transform of $g_h(t)$, the Fourier transform of $\hat{f}(t)$ is given by

$$\hat{F}(\omega) = F^*(\omega)\cdot G_h(\omega) \tag{8}$$

The zero-order hold is shown to act as a low-pass filter with a phase lag.

Without detailing the spectral characteristics of the linearly interpolated digital waveform we proceed to the case of the multiple-sampled waveform where the two successive sampling functions differ only by some delay

$$|\delta| \leq |T|.$$

SPECTRUM OF CORRELATED PROCESSING

Suppose f(t) is an analog sonar signal. By sampling with a periodic pulse signal p(t) that has period T and a finite pulse width, we get the sampled version of f(t) to be

$$f^*(t) = p(t)\cdot f(t) \tag{9}$$

Linearly interpolate between successive samples of $f^*(t)$ and then periodically sample the resultant waveform by a second sampling pulse signal q(t) that has the same period as p(t) but shifted in time by $\delta \leq T$. If h(t) represents the linearly interpolated signal, then the resulting resampled signal is given by

$$h^*(t) = q(t)\cdot h(t) \tag{10}$$

Let $F(\omega)$ is the Fourier transform of f(t) and $H^*(\omega)$ the Fourier transform of $h^*(t)$. By performing the proper manipulations in the transform domain, we get the transform $H^*(\omega)$ to be

$$H^*(\omega) = \sum_{k=-\infty}^{\infty} C_k\cdot F\left(\omega - \frac{2\pi k}{T}\right)\cdot \left[\left(1-\frac{\delta}{T}\right)e^{-j\omega\delta} + \frac{\delta}{T}e^{-j\omega(\delta-T)}\right] \tag{11}$$

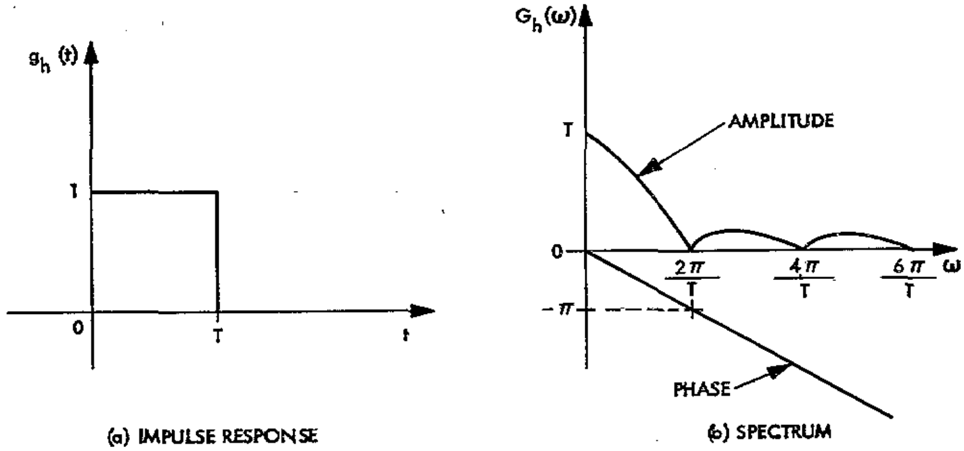


Figure 8. Characteristics of the Zero-Order Hold Interpolator

where the C_k's are the Fourier series coefficients of the finite pulse width sampling function p(t).

By comparing the spectrum $H^*(\omega)$ given by Equation (11) to that of the initially sampled function $f^*(t)$, viz,

$$F^*(\omega) = \sum_{k=-\infty}^{\infty} C_k \; F(\omega - 2\pi k/T) \qquad (12)$$

we get the spectral error function to be

$$\epsilon = (1 - \frac{\delta}{T}) \; e^{-j\omega\delta} + \frac{\delta}{T} e^{-j\omega(\delta - T)} \qquad (13)$$

Converting the error function in Equation (13) into the amplitude and phase format, and then simplifying, yields the amplitude and phase error as a function of the resampling delay δ and the frequency ω. Figure 9 shows the typical amplitude and phase error variations as a function of δ in the range [O, T], where T is the period of the master sampling function p(t).

Solving for the maximum amplitude error ϵ_A (max) and the maximum phase error ϵ_ϕ (max) in terms of frequency ω, we get the following simple expressions:

$$\epsilon_{A(\max)} = 1 - \sqrt{0.5\,[1 + \cos\,(2\pi X)]} \qquad (14)$$

$$\epsilon_{\phi\,(\max)} = \arctan\,[-\tan^3\,(\pi X/2)] \qquad (15)$$

where $X \leq \frac{1}{2}$ is the ratio of the signal frequency ω to the sampling frequency ω_s. The results are plotted in the graph in Figure 10. As expected, both the amplitude error and the phase error drop to negligible values as the frequency of the signal becomes small in comparison to the sampling frequency.

THE MEANING OF DIGITAL SOUND PROCESSING TO TRAINERS

The reliability, environmental robustness, and ever decreasing cost of digital circuitry has been

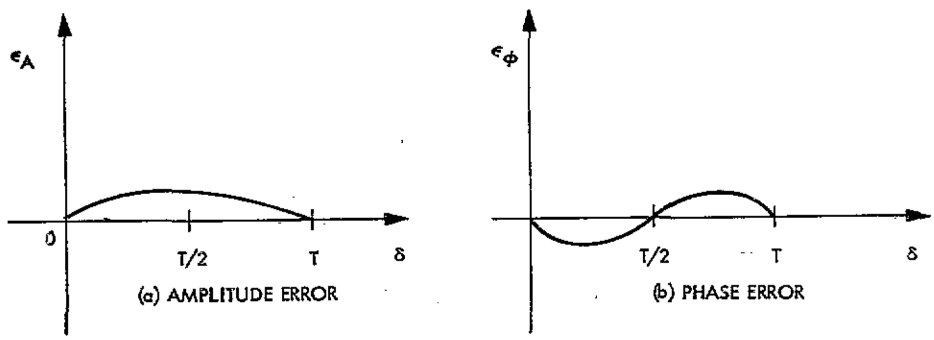


Figure 9. Error Variation as a Function of δ

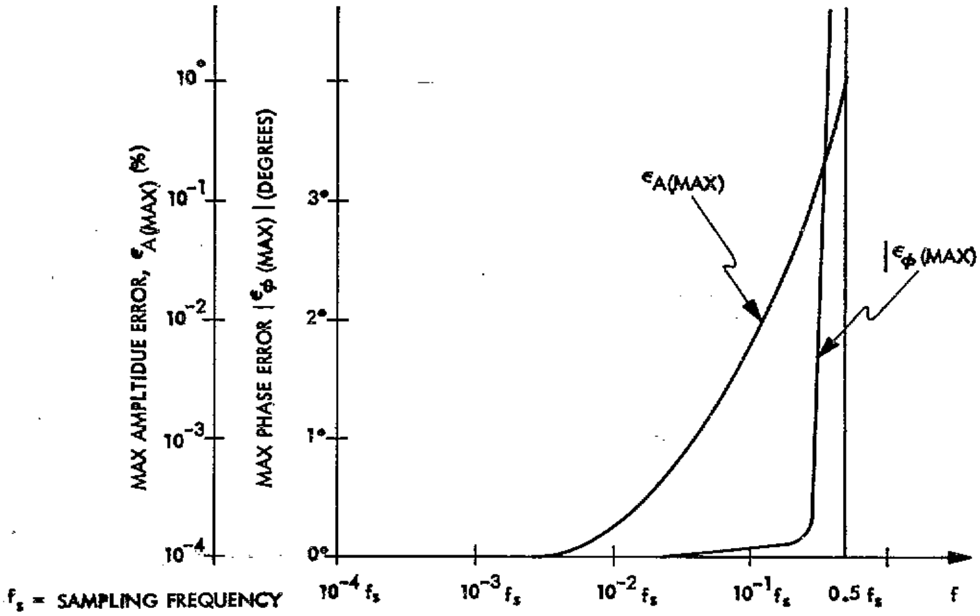


Figure 10. Maximum Amplitude and Maximum Phase Errors

well documented in the literature. As a contending technology, digital systems are replacing their analog counterparts at an ever increasing rate. Yet digital sound generation is only recently making headway in sonar stimulation design. What, then, does digital sound processing bring to trainers that analog sound processing does not?

First, digital processing brings to trainers all the advantages of digital systems, such as no temperature drift, high reliability, noise immunity and time invariability. All these advantages culminate in a device that provides long hours of service with no need to intermittently fine tune. Second, by using larger numbers of bits to represent the simulated data, finer resolution can be provided than was possible with analog circuitry. All of these advantages seem to hail digital circuitry as the answer to flawless trainer system design. In actuality, this is not completely true.

Many data processing functions, such as modulating one signal by another and multiplying one number by another, are more easily performed, and at a lower cost, by analog circuits. The only disadvantage is that these circuits suffer from the malady of temperature drifts, although this is curable at a high development and implementation cost. Analog circuit processing is also continuous data processing, which does not add spurious errors caused by sampling. The problems of aliasing due to subsampling are very definitely a digital system problem. This problem requires that sufficiently high sampling rates be used, which translates to high speed circuitry and proper filtering to discriminate against correlated noise caused by sampling and quantizing. The brute force solutions push the present technological state-of-the-art.

The need to study the effects of processing completely in the sampled domain is essential in any trainer system that is to be mechanized completely by digital circuitry. The results of this paper show the importance of spectral analysis to designing a fully digitally implemented sonar trainer. By careful design, the advantages of digital circuitry are retained without introducing the disadvantage of sampled signal representation. It is obvious that the analysis shown in this paper plays an important role in the design of proper sound signal simulators for use in sonar trainer stimulators.

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