EVALUATION OF THE EFFECTIVE BEAM GEOMETRY FOR A LASER TRANSMITTER AND A THRESHOLD DETECTOR

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ABSTRACT

During the recent development of a tactical Laser Engagement System (LES) it became necessary to accurately determine the effective beam geometry for a Gallium-Arsenide laser transmitter used in conjunction with a series of fixed threshold detectors. Of particular interest was the knowledge of the diameter of the effective detection zone as a function of range, laser power level, detector sensitivity and threshold level, laser beam divergence and atmospheric extinction. A theoretical model, based upon threshold detection at a critical irradiance level, results in a closed-form solution for the effective beam diameter as a function of all the stated parameters. The resulting equation successfully predicts the so-called "tube" effect, which has been discovered in experimental field tests, as well as the maximum effective range of the system. The equation has been programmed for the XDS Sigma 7, and computer-generated beam geometry plots are now available. The plots provide valuable system design data which has already helped to make the proposed EDM/LES more cost-effective.

SECTION I. INTRODUCTION

During the Laser Engagement System (LES) development program, a number of important system tradeoffs were required. While some of the tradeoffs could be initiated on an analytical basis, the critically important selection of the optimum number and placement of the discrete detectors on a target was performed empirically, based on experimental results from field test data. Basically, there were two reasons for this:

1. The need to insure that the performance objectives involving range, beam size, detector sensitivity, noise rejection, etc., were indeed being met under actual field conditions.

2. The fact that a detailed theoretical development of the interaction between laser output power, detector sensitivity, atmospheric transmission, beam divergence, range and effective beam "kill" diameter did not exist.

Recently, a study of the interaction of these parameters was undertaken. This study resulted in a theoretical formulation of the effective laser beam diameter as a function of the aforementioned parameters. Furthermore, an exact analytical solution of the problem resulted which included all of the relevant physics. This theoretical development has had a major impact upon design considerations for the LES engineering development model (EDM) system. Specifically it has given considerable insight into each of the following:

1. Eye safety;
2. Optimal detector placement;
3. Optimal laser power output;
4. Optimal laser beamspread;
5. Prediction of maximum range;
6. Assurance of avoiding overkill at overrange;
7. Prediction of effective beam kill diameter;
8. Prediction of effects of atmospheric absorbtion upon beam diameter and maximum range;
9. Prediction of the influence of manufacturers variations in laser output power, from laser to laser, on beam diameter and maximum range;
10. Prediction of the influence of detector/preamplifier sensitivity and threshold level upon beam diameter and maximum range.

The analysis which follows includes the derivation of the basic range/beam-diameter equation for a laser/threshold detection system.

SECTION II. DERIVATION OF THE BEAM GEOMETRY EQUATION

A matter of considerable importance throughout the LES program was the effective "kill" beam diameter as a function of: 1) Range from the laser; 2) Laser power output; 3) Laser beamspread; 4) Detector threshold; 5) Initial beam aperture diameter.

It is evident that increasing the laser output power will increase overall range of effectiveness and will also increase the effective kill diameter at a given range.

Similarly, decreasing the detector threshold will have similar qualitative effects. However, the question remains, "How does the 'kill' beam diameter depend upon these variables and what are the anticipated numerical values?"

Of particular interest are the experimentally determined facts which are:

1. The kill beam diameter for the ADM TES system tends to remain relatively constant from 100 to 300 meters. This is the so-called "kill tube" result.

2. The laser power seems to have little influence on the kill beam diameter. Laser power primarily influences maximum range.

3. The detector threshold also has a weak effect upon kill diameter.

We shall first consider a simplified model in which we neglect atmospheric losses. The radiant power in the entire beam must be the same at all axial stations since there are no losses. Hence, from the law of conservation of energy:

$$P_{tot} = \int_0^\infty 2\pi r\ H(r)dr$$

must remain invarient where H is the irradiance and r is the radial coordinate. Note that the limits of integration extend laterally to infinity to encompass all possible photons. If we assume the beam to be Gaussian then

$$H(r) = H_o\ e^{-(r/a)^2}$$

where a = the Gaussian e-folding width such that at r = a,

$$H = \frac{1}{e}\ H_o$$

and

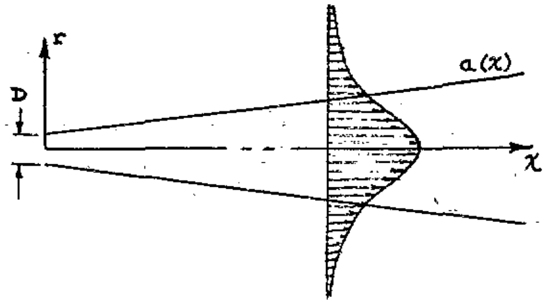
$$H_o = H_o^o \left[\frac{D}{D + X\beta}\right]^2$$

where H_o^o = centerline irradiance at exit aperture, X = o, r = o.

D = diameter of aperture (i.e., beam diameter at X = o)

X = axial coordinate

and β = beam spread in milliradians



If we define the effective beam diameter at a given range X, as the locus of positions capable of producing a given signal on the LES fixed threshold detector, this amounts to stating that $H = H_c$ where H_c = critical minimum irradiance required to establish a given threshold signal on the LES detection subsystem. Thus, we are interested in finding the locus of all points X,r such that

$$H(X,r) = H_o^o \left[\frac{D}{D + X\beta}\right]^2 e^{-(r/a)^2} = H_c$$

Note that the irradiance expression 1) Reduces to H_o^o at X = o, r = o; 2) Decreases with the inverse square of the range, X; 3) Decreases in a Gaussian fashion in the radial direction; 4) Goes to zero as $r \to \pm\infty$; and 5) Goes to zero as $X \to \infty$.

Further, we find at any axial position X = L = constant

$$P_{tot} = \int_o^\infty 2\pi r\ H(r)dr$$

$$= 2\pi \int_o^\infty H_o^o \left[\frac{D}{D + L\beta}\right]^2 e^{-(r/a)^2} dr$$

$$= \frac{2\pi H_o^o\ D^2}{\left[D + L\beta\right]^2} \int_o^\infty e^{-(r/a)^2} r dr$$

Letting $u = r^2/a^2$ then

$$r dr = \frac{a^2}{2} du$$

thus

$$\int_{r=o}^{r=\infty} e^{-r^2/a^2} r dr = \int_{r=o}^{r=\infty} \frac{a^2}{2} e^{-u} du$$

$$= \frac{a^2}{2}\left[-e^{-u}\right]_{r=o}^{r=\infty} = \frac{a^2}{2}$$

and therefore

$$P_{tot} = \frac{\pi H_o^o D^2 a^2}{\left[D + L\beta\right]^2}$$

Furthermore, since this quantity, P_{tot}, must also remain invariant at all range values, X, (neglecting atmospheric absorption) we may solve for a(X)

$$\left[a(X)\right]^2 = \frac{P_{tot_o}}{\pi H_o^o D^2} \left[D + X\beta\right]^2$$

but by definition of P_{tot_o} and H_o^o,

$$P_{tot_o} \equiv \frac{\pi D^2 H_o^o}{4} \quad \text{or} \quad \frac{P_{tot_o}}{\pi H_o^o D^2} = \frac{1}{4}$$

thus

$$\left[a(X)\right]^2 = \frac{1}{4} \left[D + X\beta\right]^2$$

$$\text{or} \quad a(X) = \frac{D + X\beta}{2}$$

Thus it is clear that the irradiance distribution

$$H(X,r) = H_o^o \left[\frac{D}{D + X\beta}\right]^2 e^{-(r/a)^2}$$

satisfies the invariance requirement as well as all five primary constraints listed above. If we now require that the "kill beam" be defined as the locus of extremal points such that $H(X,r) = H_c$, we find

$$e^{(r/a)^2} = \frac{H_o^o}{H_c} \left[\frac{D}{D + X\beta}\right]^2$$

or

$$(r/a)^2 = \ln \frac{H_o^o}{H_c} + 2 \ln \left[\frac{D}{D + X\beta}\right]$$

$$\text{or} \quad r(X) = a \left[\ln \frac{H_o^o}{H_c} + 2 \ln \left[\frac{D}{D + X\beta}\right]\right]^{1/2}$$

Thus we see that the kill radius varies as the square root of the sum of two logarithms.

Since H_c is an arbitrary kill threshold irradiance level and H_o^o is the centerline exit aperture irradiance level, which is proportional to the laser power, we may write

$$\frac{H_o^o}{H_c} = k \frac{P}{T}$$

where P = laser power

T = threshold equivalent power

k = proportionality constant

thus we may write the expression for the kill radius

$$r_{kill}(X) = a \left[\ln \left(k \frac{P}{T}\right) + 2 \ln \left(\frac{1}{1 + \frac{X\beta}{D}}\right)\right]^{1/2}$$

Note that the kill radius is a function of

1. Laser output power (P)
2. Detector threshold (T)
3. Laser aperture (D)
4. Laser beamspread (β) and
5. Range, X, from the laser.

Furthermore, note how extremely weak the various dependencies are...e.g., the square root of the logarithm of the laser power. Functionally, this explains the so-called "tube" hypothesis which has been roughly observed in field tests; namely, the fact that for a considerable range the kill diameter remains relatively constant. After some simple algebra we obtain the "Vacuum Beam Geometry Equation".

$$r_{kill} = a \left[\ln\left(k \frac{P}{T}\right) - 2 \ln\left(1 + \frac{X\beta}{D}\right)\right]^{1/2}$$

We may now evaluate some of the constants based on experimental results. Since field tests have shown that the current LES system can just barely "kill" at 400 meters range with P = 1 watt and $\beta = 2 \times 10^{-3}$ radian (i.e., the TES system kill code) then, since 400 meters = 4×10^4 cm and D = 2.5 cm

$$\frac{kP}{T} \equiv \frac{H_o^o}{H_c} = \frac{1\text{ watt}}{\frac{\pi D^2}{4}} \Bigg/ \frac{1\text{ watt}}{\frac{\pi}{4}(D + 4 \times 10^4 \times 2 \times 10^{-3})^2}$$

$$= \left(\frac{D + 80}{D}\right)^2 = \left(\frac{82.5}{2.5}\right)^2 = (33)^2$$

or

$$\frac{kP}{T} = 1090$$

∴ Since P = 1 watt and T = 10^{-3} we find k = 1.09. Thus, for the present LES system

$$\ln\left(\frac{kP}{T}\right) = \ln(1090) = 7.0$$

Hence, for the LES ADM version of the TES system the kill radius was given by

$$r_{kill} = a\left[7.0 - 2\ln(1 + 8\times10^{-2}X)\right]^{\frac{1}{2}}$$

where X is the range in meters.

Thus, we may now refer to the tabulation showing actual kill beam diameter as a function of range (Table 1).

SECTION III. ATMOSPHERIC EXTINCTION EFFECTS

The previous analysis considered the irradiance distribution resulting from a Gaussian profile laser beam in the absence of atmospheric absorption. This slightly oversimplified model is an adequate approximation for TES where the range values are relatively short. However, in the VES system the range values are considerably greater and the effects of atmospheric extinction are not negligible. Thus, we shall now consider a modification of the earlier results for beam distribution by allowing for atmospheric extinction.

From Lambert's law the irradiance distribution derived earlier may be extended to include the exponential decay resulting from atmospheric extinction in the form

$$H(X,r) = H_o^o\left[\frac{D}{D + X\beta}\right]^2 e^{-\alpha X} e^{-(r/a)^2}$$

Note that any location, X, radial integration would result in

$$P_{tot} = \frac{\pi H_o^o D^2 a^2 e^{-\alpha X}}{(D + X\beta)^2}$$

and since $a = a(X) = \frac{D + X\beta}{2}$ we obtain

$$P_{tot} = \frac{\pi D^2}{4} H_o^o e^{-\alpha X} = P_o e^{-\alpha X}$$

TABLE 1

KILL BEAM DIAMETER AS A FUNCTION OF RANGE

Range	Radius of 2 mr Beam		Kill Radius	Kill-Beam Diameter	
X	a	r / a	r	d = 2r	d (inches)
0M	1.25 cm	2.65	3.31 cm	6.62 cm	3.61
1M	1.35 cm	2.62	3.54 cm	7.08 cm	3.79
10M	2.25 cm	2.42	5.44 cm	10.88 cm	4.28
25M	3.75 cm	2.20	8.25 cm	16.50 cm	6.50
50M	6.25 cm	1.96	12.25 cm	24.50 cm	9.65
100M	11.25 cm	1.62	18.20 cm	36.40 cm	14.33
200M	21.25 cm	1.16	25.00 cm	50.00 cm	19.69
300M	31.25 cm	0.75	23.50 cm	47.00 cm	18.51
350M	36.25 cm	0.32	11.70 cm	23.40 cm	9.27
400M	41.25 cm	0	0	0	0

which clearly decreases with range in the usual exponential manner. Hence, using the modified irradiance expression, allowing for atmospheric absorption, and employing the criteria of critical irradiance we desire the locus of positions such that $H(X,r) = H_c$

$$H_c = H_o^o \left[\frac{D}{D + X\beta}\right]^2 e^{-\alpha X} e^{-(r/a)^2}$$

or

$$e^{(r/a)^2} = \frac{H_o^o}{H_c} \left[\frac{D}{D + X\beta}\right]^2 e^{-\alpha X}$$

or

$$\left(\frac{r}{a}\right) = \ln \frac{H_o^o}{H_c} - 2 \ln \left[1 + \frac{X\beta}{D}\right] -\alpha X$$

or

$$r(X) = a(X) \left[\ln \frac{H_o^o}{H_c} - 2 \ln\left(1 + \frac{X\beta}{D}\right) - \alpha X \right]$$

which is very similar to the earlier results except that the bracket is now modified by the $-\alpha X$ term. This is the complete "Beam Geometry Equation."

Note:

1. Atmospheric extinction will always tend to reduce the effective beam diameter.

2. Atmospheric extinction will only be significant when either the range is great or the atmospheric extinction coefficient, α, is quite large.

For example, taking the case $\lambda = 0.904\mu$, (the GaAs laser) standard clear conditions, at sea level, we find:

$$\alpha = 1.2 \times 10^{-4} M^{-1}$$

and the equation for TES kill radius becomes

$$r_{kill} = a\left[7.0 - 2 \ln (1 + 0.08X) - 1.2 \times 10^{-4} X\right]^{\frac{1}{2}}$$

and for X = 400 meters we see that the last term amounts to only 4.8×10^{-2} relative to 7.0 for the first term. Thus, atmospheric absorption will have a very small effect upon kill beam diameter for the TES system.

However, let us now consider the VES system. Here we find

$P = 5.0$ watts

$\beta = 1.0 \times 10^{-3}$ rad (kill beam)

$D = 2.5$ cm

$T = 0.50 \times 10^{-3}$ watt (more sensitive preamp)

$k = 1.09$

thus

$$\frac{kP}{T} = \frac{1.09 \times 5}{0.50 \times 10^{-3}} = 10{,}900$$

$$\ln_e \left(\frac{kP}{T}\right) = \ln_e (10{,}900) = 9.28$$

Thus, for the present VES system we obtain the maximum range, X max at that location where $r_{kill} = 0$. From the Beam Geometry Equation we see that when r = 0

$$\ln\left(\frac{kP}{T}\right) = \ln \left[\left(1 + \frac{X\beta}{D}\right)^2 e^{\alpha X}\right]$$

or

$$\frac{kP}{T} = \left(1 + \frac{X\beta}{D}\right)^2 e^{\alpha X}$$

or

$$\left(1 + \frac{10^{-3} X}{2.5 \times 10^{-2}}\right)^2 e^{1.2 \times 10^{-4} X} = 10{,}900$$

or

$$(1 + 4 \times 10^{-2} X)^2 e^{1.2 \times 10^{-4} X} = 10{,}900$$

This is a transcendental equation not readily solved for X. A numerical solution is shown in Table 2.

Xerox Electro-Optical Systems has recently written a computer program to allow rapid parametric solution of this equation, as well as automatic computer plotting. Two additional modifications from the original analysis were made in the computer program. The first is to account for the non-circular beam cross-section which results from the different meridian polar distribution of the GaAs lasers. They are found to have somewhat elongated beam shapes. A reasonable

TABLE 2

Range X	$(1 + 4 \times 10^{-2} X)^2$	$e^{1.2 \times 10^{-4} X}$	$(1 + 4 \times 10^{-2} X)^2 \; e^{1.2 \times 10^{-4} X}$
500M	440	1.06	456
700M	840	1.087	912
1000M	1680	1.127	1,892
1200M	3720	1.197	4,450
2000M	6520	1.272	8,330
2200M	7890	1.301	10,280
2250M	8280	1.310	10,850

approximation to the beam shape cross-section in a plane perpendicular to the optical axis is an ellipse. If the semi-major and semi-minor axes of the ellipse are denoted by "a" and "b", respectively, then the area of the ellipse is simply πab. If we assume that the same total energy passes through the ellipse as would have passed through the circle of radius r_{kill}, calculated previously then we find

$$A_{beam} = \pi r_{kill}^{2} = \pi ab$$

or

$$r_{kill}^{2} = ab$$

Since the ratio $b/a = n$ may be determined by best fit with experimentally measured values, then we may solve for

$$a = (n)^{-\frac{1}{2}} r_{kill}$$

and

$$b = (n)^{\frac{1}{2}} r_{kill}.$$

The second modification incorporated a refinement in the computer program of the detector threshold parameter. The newer threshold was based on limited empirical test data of a few ADM systems.

Plots of the kill radius versus range for both the major and minor meridians are shown. Figure 1 shows the computer generated results for the predicted characteristics of the ADM TES kill beam. The outline of a human figure is shown, to scale, to allow the reader to obtain a physical feeling for the beam size. A number of important results are evident upon inspection of these results:

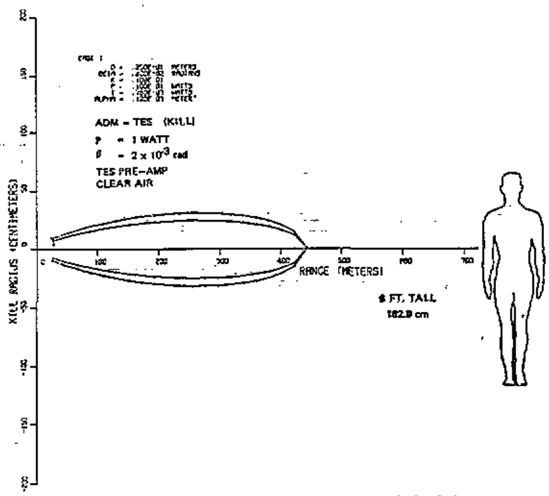


Figure 1. ADM TES Kill Beam Shape Versus Range - Clear Day

1. The theoretical confirmation of the experimentally determined "kill tube" effect. Namely, the fact that there is relatively little variation in effective kill diameter from 100 to 400 meters.

2. The abrupt decrease in diameter beyond 400 meters range, leading to absolutely no possible kill beyond 440 meters.

3. The major and minor meridian effects which have also been observed experimentally.

4. Prediction of high probability of kill at 300 meters if aimed within a 30 cm (i.e., 1 foot) radius of a detector. This has been verified

5. Prediction of essentially zero probability of kill at 450 meters. This has also been experimentally verified.

Figure 2 shows the same situation, except that rather than "standard clear" atmospheric conditions, we now assume haze conditions corresponding to about 8 kilometers visibility. Note that the results are changed very little. The maximum range is now 420 meters instead of 440 meters, and the maximum effective kill diameter is now about 64 cm compared to 67 cm for standard clear conditions. Also plotted on the graph of Figure 2 are data points reflecting actual tests at 300 meters using the ADM TES equipment, which were conducted on a "hazy" day.

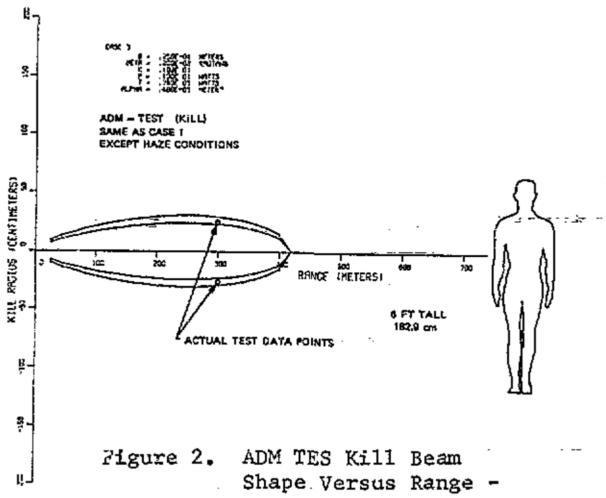


Figure 2. ADM TES Kill Beam Shape Versus Range - Hazy Day

As a final example of the sensitivity of the analysis, Figure 3 shows the same result except for $\alpha = 0$ (i.e., perfect transmission through a vacuum). While this cannot, of course, occur on the earth's surface, it is clearly a limiting case for maximum possible range. We see that this gives a maximum range of 450 meters, and a maximum kill diameter of 68 cm. Obviously, the effects of atmospheric extinction from $\alpha = 0$ to $\alpha = 1.2 \times 10^{-4} M^{-1}$ (standard-clear) are very small. This result in effect guarantees that by proper selection of laser power, beamspread, detector, threshold, and detector sensitivity, it is possible to design a LES system which cannot produce overkill-at-overrange. This is a very important result. It is now possible, in effect, to "work the problem backwards." That is, starting from the maximum overkill range, one can calculate those values of P, β, k and T, which will result in a maximum range just short of the maximum allowable overkill range under vacuum conditions. One is then guaranteed not to exceed this range for <u>any</u> real atmospheric conditions.

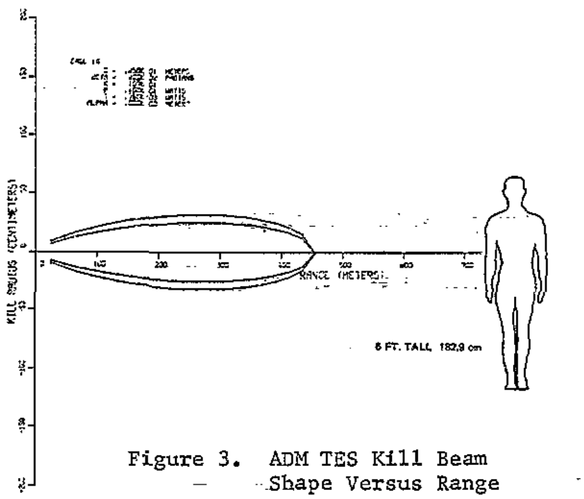


Figure 3. ADM TES Kill Beam Shape Versus Range If Operating in a Vacuum

Examples of calculations pertinent to the ADM VES are shown in Figures 4 and 5, for standard-clear and haze conditions, respectively. Note that the curves predict good kill probability at 2000 meters under standard clear conditions, but a maximum range of about 1900 meters under haze conditions. The values of P, β, k and T had been chosen for ADM VES prior to the theoretical development of the "Beam Geometry Equation." With our present computer capability, we are now able, as stated above, to "work the problem backwards" and evolve results for EDM VES as shown in Figure 5. Here it can be seen that the proper choice of parameters does indeed insure:

- No overkill-at-overrange.
- Good kill probability at 2000 meters in standard clear or hazy atmospheric conditions.
- Increased beam diameter (252 cm maximum for EDM versus only 184 cm for ADM), which implies fewer detectors are required and, thus, reduces system cost.

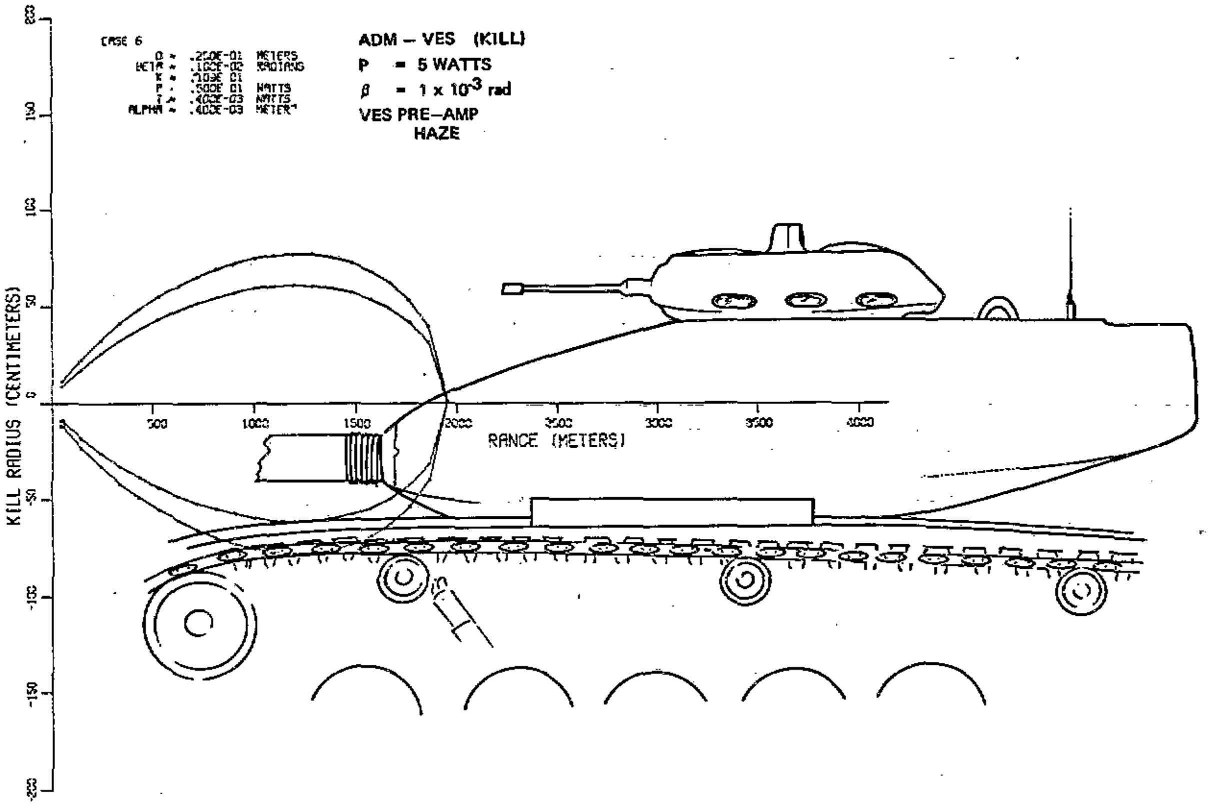


Figure 4. ADM VES Kill Beam Shape Versus Range - Hazy Day

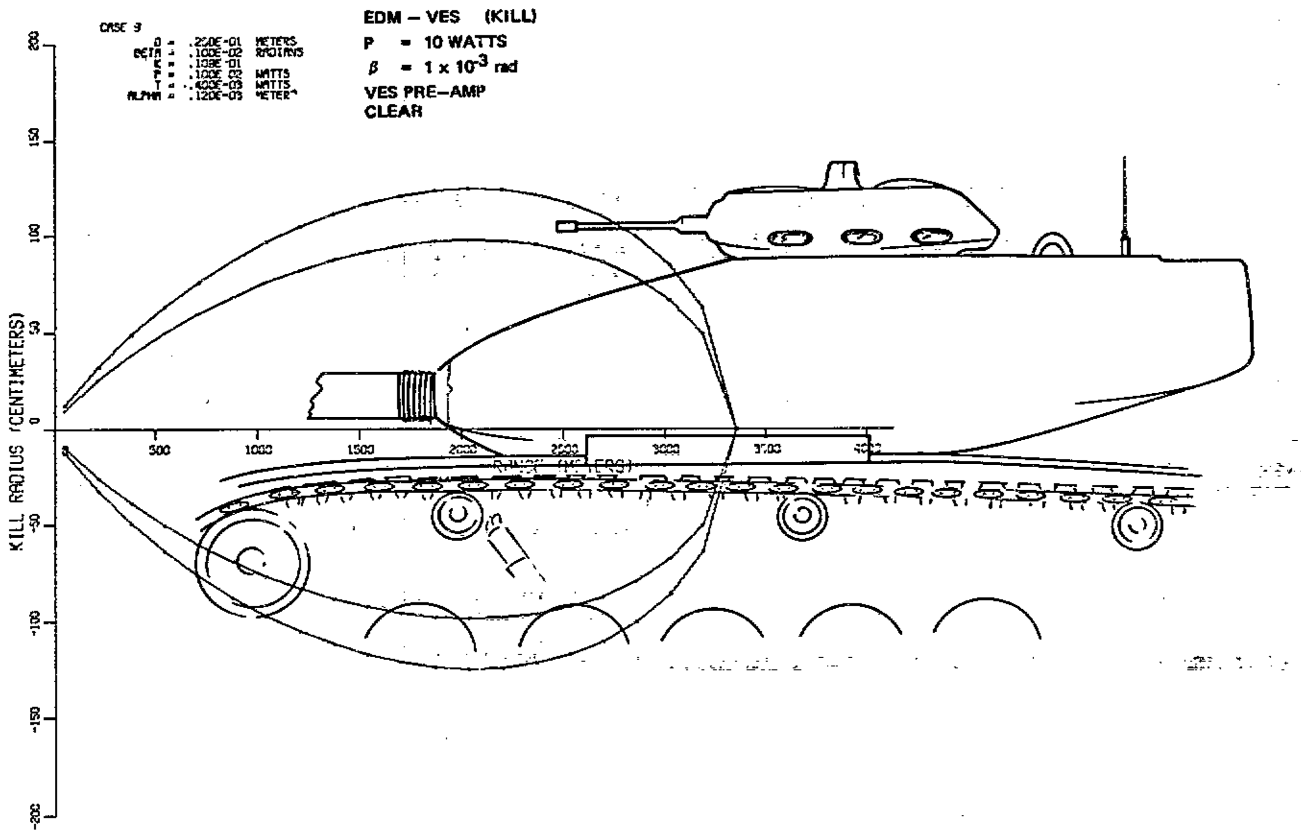


Figure 5. EDM VES Beam Shape Versus Range - Clear Day

Similarly, Figure 6 is the result of a calculation appropriate to the suggested goal for the proposed EDM version of MILES for the M60 machine gun, where a new maximum range of 600 meters might be desired. By proper choice of P, β, k and T, we have

- Met the maximum range goal.
- Retained good kill diameter.
- Continued utilization of low cost solar cell detectors.
- Insured good "kill" probability to maximum range.
- Insured no overkill-at-overrange.
- Remained eye safe at all ranges.

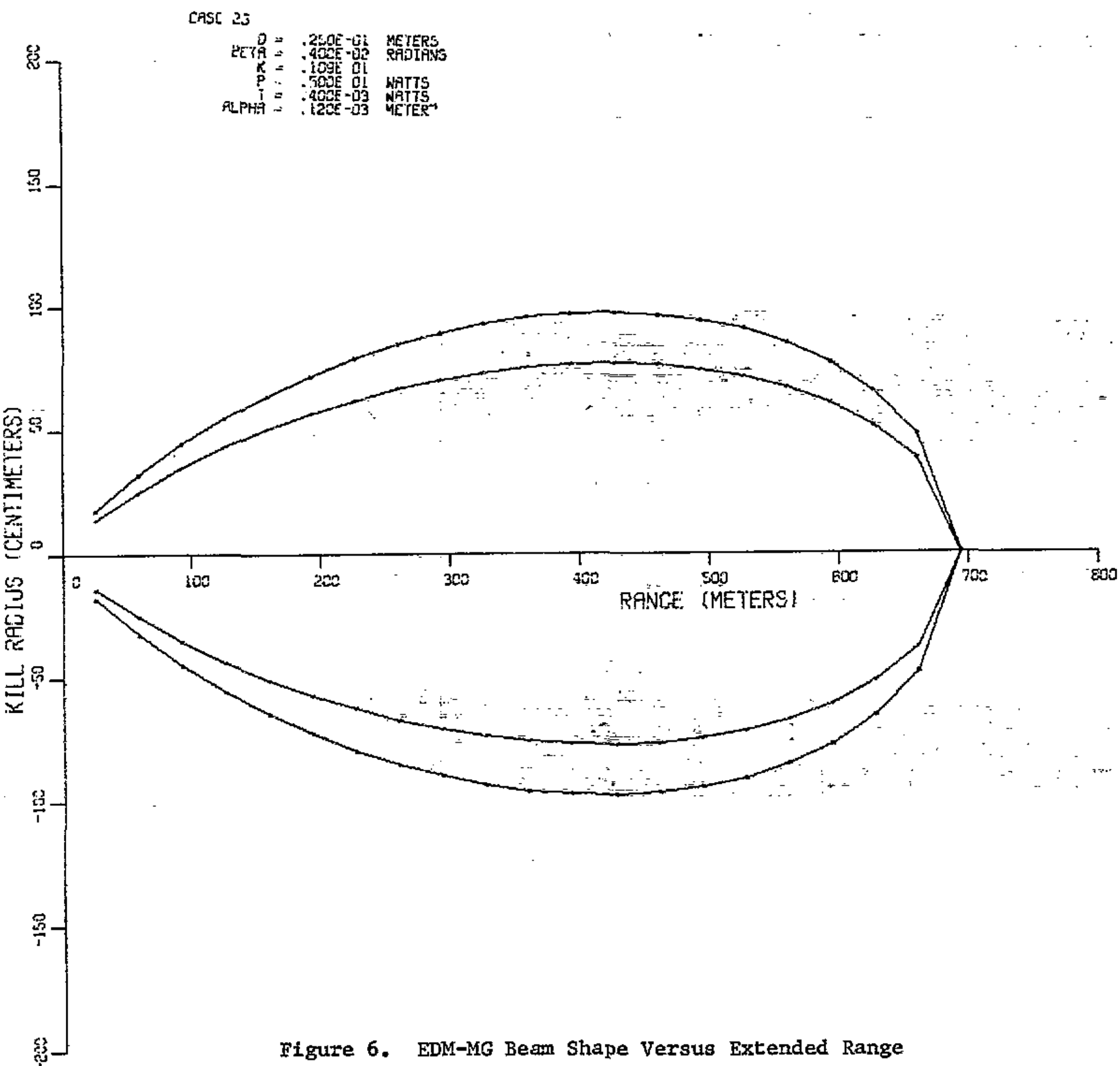


Figure 6. EDM-MG Beam Shape Versus Extended Range

ABOUT THE AUTHOR

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