Application of GPS to Hybrid Live/Constructive/Virtual Training Systems

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ABSTRACT

GPS user equipment has matured and is now available to support the use of live players in networked live/constructive/virtual wargaming simulations. GPS provides true WGS-84 based coordinate information anywhere in the world at any time and to accuracies at the 5 ft (1σ) level (demonstrated in high dynamic aircraft using differential GPS).

In supporting DIS-based hybrid live/constructive/virtual networked team training, GPS is directly applicable to the dead reckoning requirements of DIS. The on-board state vector for an integrated GPS/Inertial Reference Unit provides accurate position, velocity and acceleration as well as attitude and attitude rate information so that dead reckoning thresholds can be both position and attitude driven. A simplified analysis is presented in the paper to derive dead reckoning update rates from the G loading levels of various player dynamics. Also, information is provided which results in word length requirements for GPS-based state vector information for transmission over minimum word length DIS Field Instrumentation Protocol Data Units (PDUs, which are the data block formats). The coordinate frame problem in use of GPS-based state vector information from fixed ranges is also addressed, showing that the use of a local geodetic frame is preferable to the use of an earth centered earth fixed frame, in that it is more efficient of network PDU word length. Weapon scoring requirements using GPS-based state vectors are addressed in terms of GPS state vector accuracy required to score various weapons and provide "positive training". These requirements are all applicable to the JTCTS and NGTCS programs which are in the formative stages and will use GPS-based information in DIS Field Instrumentation PDUs.

Results are presented of a combined Northrop/IEC demonstration using the China Lake RAJPO GPS assets linked into a DIS demo for I/ITSEC in November 1993.

ABOUT THE AUTHORS

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INTRODUCTION

The Global Positioning System (GPS), which is now operational, consists of a network of 24 satellites which provide true WGS-84 based three-dimensional position and time on a continuous basis with world-wide coverage[1,2,3]. Although the overall and usually quoted GPS position accuracy is 16 meters (52.5 feet) SEP (Spherical Error Probable), experience in most applications has been considerably better than this. During the Desert Storm operation from 15 January to 3 March 1991, long term averages over 11,000 navigation solutions showed the average SEP to be 8.3 meters[3] (27.2 feet), rather than the 16 meter specification. With differential GPS[4], which uses a ground reference receiver to remove common mode errors between the reference station and the user (such as satellite orbit and clock errors, and the common mode portion of ionospheric errors), errors of 2 meters or less are routinely achieved.

GPS therefore provides an invaluable tool in instrumenting live platforms on training ranges. In addition to the high accuracies, operation is achieved on high dynamic platforms at G loads to 8 Gs. For high dynamic platforms, the GPS receiver is best integrated with an inertial reference unit. GPS and inertial systems are highly synergistic; the GPS removes the troublesome biases of low cost/low quality inertial systems, and the inertial system carries the GPS receiver through high dynamic maneuvers and temporary signal blockages caused by shadowing of the antennas.

Typical accuracies of differential GPS in a high dynamic platform using method one[4], which is by far the best because it accounts for rapid satellite switching in high dynamic maneuvers, are shown in figure 1, which shows horizontal and vertical position and velocity errors in both F-15 and F-16 flight tests[5]. These particular flight tests were conducted by the Tri-Service GPS Range Applications Program Program Office (RAJPO) on the High Dynamic Instrumentation Set (HDIS) developed for that program by Interstate Electronics Corporation.

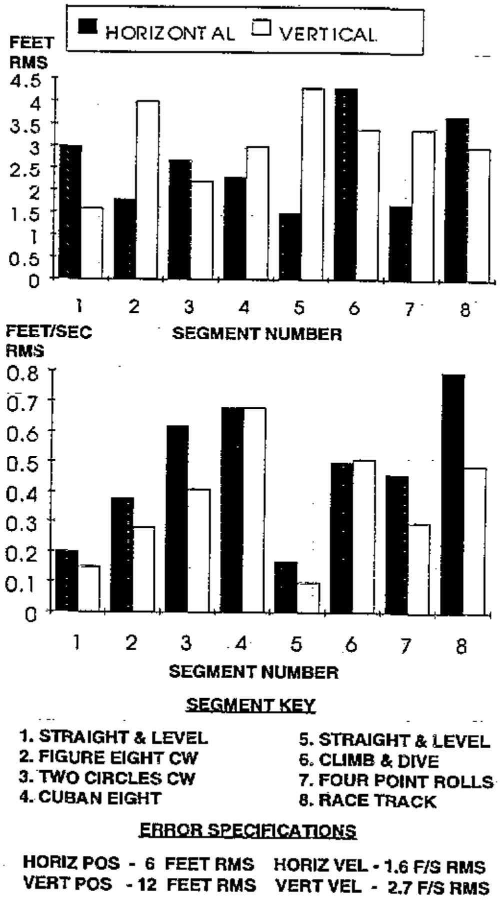
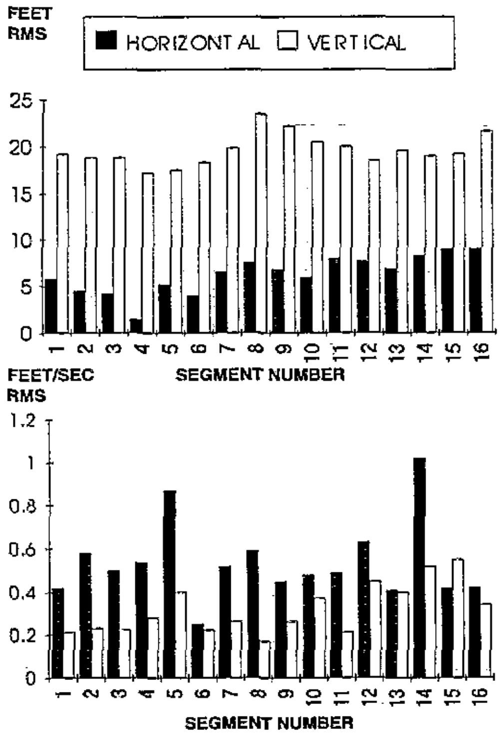


Figure 1. Flight Test Results[5] for differential GPS with inertial aiding

Absolute accuracy test results are shown in figure 2. The dynamics in these tests covered the full range up to 8 Gs in the various maneuvers used with intermittent satellite visibility caused by antenna shadowing. Ground truth was provided to 2 ft. accuracy at Eglin Air Force Base by a truth system consisting of cinetheodolites, laser ranging, FPS-16 radars and aircraft inertial navigation systems.



SEGMENT KEY

1. STRAIGHT & LEVEL	9. STRAIGHT & LEVEL
2. FIGURE EIGHT CW	10. STRAIGHT & LEVEL
3. TWO CIRCLES CW	11. FIGURE EIGHT
4. STRAIGHT & LEVEL	12. TWO CIRCLES CW
5. RACETRACK CCW	13. STRAIGHT & LEVEL
6. STRAIGHT & LEVEL	14. RACETRACK CCW
7. FIGURE EIGHT CW	15. STRAIGHT & LEVEL
8. TWO CIRCLES CCW	16. STRAIGHT & LEVEL

ERROR SPECIFICATIONS

HORIZ POS - 21 FEET	HORIZ VEL - 1.6 F/S
VERT POS - 35 FEET	VERT VEL - 2.7 F/S

Figure 2. Flight Test Results[5] for absolute GPS with inertial aiding

APPLICATION TO TEAM TRAINING

The way in which GPS will be used in large-scale training systems is currently being defined by key programs such as JTCTS and MAIS. An effort to develop standards for these applications is being carried on by the Field Instrumentation Working Group which is a part of the Distributed Interactive Simulation (DIS) standards development activity. Currently, a draft standard for field instrumentation use has been prepared and is in review. The key driver in developing these standards is minimizing the data that needs to be sent on the data link, since the data link has proven to be the main bottleneck in instrumenting large training exercises.

DIS Dead Reckoning Algorithms

A key part of the DIS standards is a *technique which is very useful with regard to* efficiently loading a data link with time-space-position information (TSPI). This algorithm is shown in the block diagram of figure 3. The GPS receiver output, integrated with an inertial reference unit so it periodically measures both position and attitude, is compared with the output of a "dead reckoning model". This model is a time extrapolation of previous outputs of the GPS receiver ($x = x_o + v_o t + a_o t^2$).

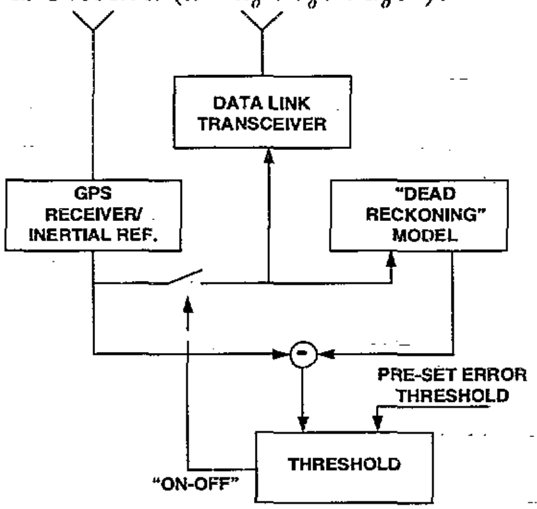


Figure 3. Dead Reckoning Algorithm for efficiently loading a data link with TSPI

When the error between the GPS-measured position (and attitude) and the extrapolated position (and attitude) exceeds some threshold level (like 10 or 20 ft.), the GPS measured state is input to both the data link and the dead reckoning model. At the distant data link receiver terminal, a similar dead reckoning model is also updated with a new state vector. The two dead reckoning model errors are thereby corrected and they can then run for a while at acceptable error levels until again updated. The algorithm thus provides a means of minimizing data link loading, according to the activity level of the aircraft. When the aircraft is flying with minimal acceleration, the extrapolation can run much longer than when it is going through high-dynamic maneuvers. It therefore provides a means of taking advantage of the fact that most of the aircraft in an exercise are not maneuvering, and can be sampled at a low rate, and those that are in high dynamic maneuvers are automatically sampled at a high rate.

This algorithm is ideal for training systems because it allows supporting very high fidelity weapon simulations by closing down the error threshold for certain exercises requiring high precision such as no-drop-bomb-scoring (NDBS). It is even possible to close some portions of the error down more than others, such as closing down the vertical error more than others for NDBS, since NDBS is particularly sensitive to vertical error. The possibility also exists of closing the error threshold down only when required, such as during weapon launch, and when the aircraft is paired as a target.

One of the key questions with regard to the use of this algorithm in a large scale training system is how fast it will update vs. platform dynamics and threshold level settings. The answer to this question heavily influences data link requirements, and also affects instrumentation accuracy levels. In order to provide an easily analyzable case to demonstrate results and to provide some upper bounds on update time or "rules of thumb" as a guide for system design, the aircraft (or other platform) can be assumed to be flying in circular turns at a constant G loading. Sections of circular turns are common in aerobatic maneuvers. Even though a complete circle is not flown, small sections of circles are representative of sections of aerobatic maneuvers, since aircraft are constrained by the laws of physics to fly in a circular path constrained by G loading. More complex maneuvers can be considered as being made up of sections of circular turns. Other platforms, such as ships, also commonly make circular turns.

By using this simple maneuver as a basis for analysis, it is possible to easily derive the upper bounds for update times for various dead reckoning models versus their G loading and threshold values. In the case of aircraft (and ships), the roll angle is ignored in this analysis, and only pitch and yaw angles are considered. Also, angle-of-attack and angle-of-sideslip are assumed constant, such as would occur in a steady-state circular turn. Although these assumptions may appear to limit the validity of the analysis, they allow arriving at first-order approximations which have been shown to be consistent with flight simulator test results, although somewhat optimistic, since they are more in the nature of bounds.

Using this approach, these update times were calculated as outlined in appendix A and are presented in figure 4 for various G loadings, position thresholds and attitude thresholds.

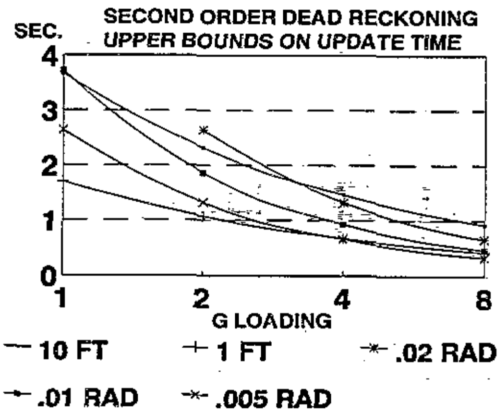


Figure 4. TSPI Update Time Bounds for Position and Attitude Threshold Second Order Dead Reckoning

An interesting comparison of these results can be made with data from the Northrop Corporation Flight Simulation Laboratory[6]. Using thresholds of 9.1 feet in position, 3 degrees in attitude, and second order dead reckoning, they obtained an average of about 1.2 updates per second for second order dead reckoning in high dynamic air combat scenarios, and an average of 2.42 updates per second for the same scenarios with first order dead reckoning.

It would be beneficial to the field instrumentation community if further similar studies were done by organizations doing dead reckoning in large scale simulations. This would allow further refinement of these bounding update time levels.

TSPI Word Length Requirements
Considerable effort has been expended by the Field Instrumentation Working Group of the DIS in the past two years in attempting to reduce the PDU (Protocol Data Unit) sizes for field instrumentation. The standard DIS PDUs are much too large to be feasible for range data link use. For example, the standard DIS entity state PDU, which defines the state vector of a platform, contains 64 bit position and velocity words. In this regard, the requirements of integrated GPS/Inertial systems for word length should be taken into account, because these are the TSPI sensors that will be used for the indefinite future in these systems. As shown in the above test data, instrumentation accuracies of close to 1 foot in position and close to 0.1 ft/sec in velocity are achievable. There is then no point in much more word length in the PDUs than that which will support 1 foot instrumentation. Attitude accuracies of 0.1 to 0.2 degrees are achieved by these same instrumentation systems, which should set the resolution of the attitude information in the DIS PDUs. Assuming maximum values such as would be required for a training system such as TCTS, the word lengths shown in table 1 result from these instrumentation accuracy levels. It should be noted that these word lengths are much less than those in the current DIS standard.

The Field Instrumentation Working Group of the DIS has proposed a flexible format of "Profiles" for the field instrumentation PDUs[7] which would have the flexibility to support variations in applications dictated by various ranges. This would be implemented by table-driven software where the profiles for each application provide the control for parsing the PDU data. With this approach, the use of word lengths no longer than the instrumentation will support should be possible.

Table 1. TSPI Word Length Requirements for a typical range training application

Full Scale Values	Resolution of LSB	Word Length Required
X,Y - 2761 nmi. =16777216 ft.	1 foot	24 bits
Height - 65536 ft.	1 foot	16 bits
Velocity - +/-3277 ft/sec	0.1 ft/sec	16 bits
Acceleration - +/- 10 Gs	300 uG ~ .01 ft/sec2	16 bits
Attitude - 360 degrees	0.05 degree	13 bits

Coordinate Frames for TSPI
The choice of coordinate frame for use in the field instrumentation PDUs also affects the number of bits required in the PDUs. For a purely surface-based exercise, there is no need to send height information, since the position coordinates provide all of the required information. Even for aircraft, fewer bits are required for instrumenting height above a range than the horizontal dimensions of the range, as seen in table 1. In comparing the use of ECEF coordinates with geodetic coordinates, studies have shown[8] that for a hypothetical 155 x 155 NM x 65000 ft range considered, 55 bits are required (for 4 ft resolution) for the three coordinates of position for translated ECEF coordinates, regardless of whether 2-dimensional or 3-dimensional information is required. For translated geodetic coordinates, 51 bits are required for the three position coordinates if height information is required (3-dimensional case), and only 37 bits are required if height information is not required (2-dimensional case). In the 2-dimensional case, velocity and acceleration state bit requirements are also significantly reduced when geodetic coordinates are used, since no height terms are necessary. This is not to say that ECEF or some other coordinate frame is not used internally for computation. ECEF coordinates are simply not the most efficient frame to use for data transmission.

Weapon Scoring for positive training
In applying GPS to training such as an ACMI system, where the positions of the shooter and target, and the attitude of the shooter must be instrumented to provide information to weapon simulations, it is necessary to define TSPI accuracies (this determines

whether absolute or differential GPS is required) for the GPS instrumentation system which will result in positive training. There is no clear definition of what constitutes positive training. For example, if guns are to be used which require 0.1 degree attitude accuracy in the shooting platform, positive training for gun scoring cannot result if only a 2 degree instrumentation system is used. There might as well not be any instrumentation because the scoring can be predicted as well by a coin toss.

There appear to be no guidelines currently to define what TSPI accuracies are required. A suggested approach for unguided ballistic weapons is contained in Appendix B. It is hoped that the training community will soon address this problem, and in addition the corresponding case for guided weapons.

Demonstrations
In November 1993, a demonstration of the use of GPS was conducted by the Northrop Corporation and IEC, using the RAJPO GPS assets at the China Lake Naval Weapons site. Data from the China Lake site was sent by phone line to the Northrop facility at Hawthorne, California, where dead reckoning was applied to the data. Also additional simulated aircraft were flown with the live aircraft in the same scenario. This was an initial demonstration of the use of live and constructive platforms together. It was demonstrated at the 1993 I/ITSEC.

CONCLUSIONS

The GPS satellites are now in place, the user equipment is available, and the DIS is proceeding to define the Field Instrumentation PDUs. The pieces are coming together for the next generation of live/constructive/virtual training systems.

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APPENDIX A
TSPI UPDATE TIME BOUNDS FOR DEAD RECKONING

To derive the update times for the dead reckoning algorithm, the aircraft is assumed to be flying at a constant airspeed, and to be making turns having various G loadings. In these circular turns, assume the position of the aircraft to be described by the vector

$$\bar{r} = R(\cos \omega t \, \bar{i} + \sin \omega t \, \bar{j}) = R\bar{\rho}$$

where $\bar{r} =$ aircraft position vector
$\bar{\rho} =$ unit radius vector
$\bar{i}, \bar{j} =$ unit x and y vectors
$R =$ turning radius
$\omega =$ rate of turn (radians/sec)

The velocity vector is the derivative of this, or

$$\bar{\upsilon} = \omega R(-\sin \omega t \, \bar{i} + \cos \omega t \, \bar{j}) = V\bar{\nu}$$

where $\bar{\nu} =$ unit velocity vector
$V = \omega R$

The acceleration is, by differentiating again,

$$\bar{\alpha} = -\omega^2 R(\cos \omega t \, \bar{i} + \sin \omega t \, \bar{j})$$
$$= -\omega^2 R\bar{\rho} = -A\bar{\rho}$$

where $A = \omega^2 R = V^2 / R$

The jerk or next derivative is needed, because it is the lowest order rate which is not measured, and therefore contributes the most error. It is

$$\bar{\varsigma} = \omega^3 R(\sin \omega t \, \bar{i} - \cos \omega t \, \bar{j})$$
$$= \frac{-A^2 \bar{\nu}}{V} = -J\bar{\nu}$$

The position error due to the jerk will then be

$$\varepsilon = \frac{J}{6}\Delta t^3 = \frac{A^2}{6V}\Delta t^3$$

We can then calculate the update time that the TSPI algorithm will operate at as a result of the position error to be approximately (setting the threshold equal to the position error)

$$\Delta t = \sqrt[3]{\frac{6\varepsilon V}{A^2}}$$

The algorithm of figure 3 also can operate on an attitude error threshold. In the DIS dead reckoning approach, a combined position and attitude error threshold is used. If either position or attitude error exceeds thresholds, the dead reckoning model is updated.

The attitude of the aircraft is constantly changing in the turn and is represented as the unit velocity vector

$$\bar{\nu} = -\sin \omega t \, \bar{i} + \cos \omega t \, \bar{j}$$

The attitude rate is the derivative of this, or

$$\bar{\psi} = -\omega(\cos \omega t \, \bar{i} + \sin \omega t \, \bar{j})$$

The second derivative of attitude is then

$$\bar{\mu} = \omega^2(\sin \omega t \, \bar{i} - \cos \omega t \, \bar{j}) = -\frac{A^2}{V^2}\bar{\nu}$$

If the attitude threshold is then θ_t,

$$\theta_t = \frac{1}{2}\frac{A^2}{V^2}\Delta t^2$$

the update time that the TSPI algorithm will operate at as a result of the attitude error will be approximately (setting the threshold equal to the attitude error)

$$\Delta t = \sqrt{\frac{2\theta_t V^2}{A^2}}$$

The update time for the combined position and attitude error threshold criteria will then be the smaller of the two or

$$\Delta t = Min\left[\sqrt[3]{\frac{6\varepsilon V}{A^2}}, \sqrt{\frac{2\theta_t V^2}{A^2}}\right]$$

These update times are calculated and plotted in figure 4 for various G loadings, position thresholds and attitude thresholds.

This approach allows a quick comparison to be made of first and second order dead reckoning. If only position and velocity are used (first order), the acceleration is not instrumented. The position error due to the acceleration not being instrumented is then

$$\varepsilon = \frac{A}{2}\Delta t^2$$

The update time that the TSPI algorithm will operate at as a result of the position error is then approximately (setting the threshold equal to the position error)

$$\Delta t = \sqrt{\frac{2\varepsilon}{A}}$$

To compare this first order dead reckoning model with the second order model, take the case of a 5 ft. threshold at 8 Gs. For the first order case shown here, the result is 0.198 seconds, as compared to 0.728 seconds for the second order dead reckoning model. This demonstrates the value of the second order model in reducing the sample rate required.

The use of circular turns at constant G loading provides a simple and convenient analysis tool for dead reckoning models. Update time bounds for specific G loads can be analytically determined, and can be verified by simulation or flight test. With this, the improvement of the second order dead reckoning model can be demonstrated analytically.

APPENDIX B
TSPI ACCURACY REQUIREMENTS FOR UNGUIDED WEAPON SCORING

Introduction

The purpose of this appendix is to outline a means of evaluating instrumentation accuracy requirements for unguided ballistic weapon scoring in simulated weapon delivery training exercises. In evaluating scoring effectiveness, the effectiveness of the following are considered:

- The effectiveness of the particular weapon against the target of interest, given perfect performance by the operator of the weapon system.

- The effectiveness of an instrumentation system in scoring simulated weapon firings of the weapon against the target of interest.

- The scoring of the trainee in firing the weapon against the target of interest.

The first parameter of importance relates to target size and other characteristics, as they relate to the weapon used against it. This includes target overall dimensions as well as particular areas of the target that are most vulnerable to the weapon. For example, this implies the dimensions of the engine room area of a ship, and its vulnerability to a bomb or torpedo.

In this discussion, we roll all of these target size /weapon effectiveness characteristics into effectively an increased target size with a dimension we call t, as shown in figure B-1. This target size parameter must be determined from the combination of target and weapon characteristics as outlined above, and may also be dependent upon target orientation.

The second parameter deals with weapon dispersion effects that are uncontrollable by the trainee. These effects should be distinguished from the dispersion effects that the training program is attempting to reduce. In dropping dumb bombs, this parameter should include unknown dispersing effects, or effects that the trainee is not expected to account for, such as wind changes (if he is accounting for wind in some way, such as in high altitude bombing), local gravity anomalies (again this probably only applies to very high altitude bombing and to ballistic missiles). When an automatic bomb release system is used, the errors of the automatic system should be included here, since they represent errors that would occur even if the trainee performed his procedure perfectly.

Uncontrolled dispersion effects are then rolled up in this parameter and called σ_d, with dimensions the same as target size t. This is illustrated in the diagram of figure B-1.

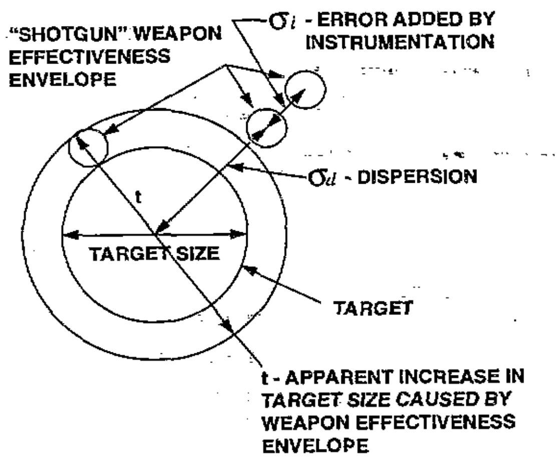

Figure B-1. Target Size and Error Definitions

Next are the instrumentation errors in instrumenting the trainee. These are termed σ_i. This term includes the uncertainty in both target and shooter position and attitude (and how their derivatives integrate into position and attitude error), since relative position and attitude are the parameters of interest.

The last parameter is the dispersion measure of the trainee. With a gun, this is his aiming error when firing, and is the principle measure of his training program with that weapon. Since instrumentation error is the main interest here, the error of the trainee is not discussed further. That is the subject of the training program which uses the instrumentation system.

Given these parameters, an analysis can be performed to determine various probabilities. The probabilities of interest are:

- The probability of a hit (P_h) given perfect performance by the trainee. This is dependent upon the first two measures t and σ_d. The probability of a hit is the probability that the weapon will hit inside the target dimension area including the effective size increase caused by the weapon effectiveness envelope (the trainee doesn't need to be so accurate if the weapon effectiveness envelope is large). It can be argued that a training program gives realistic, positive training if the probability of a hit and miss match the probability obtained if the trainee performs perfectly. These first probabilities are a measure of effectiveness of the weapon against the target of interest.

- The probability of a hit (P_{hi}) when the instrumentation system is added (includes σ_i as well as t and σ_d) is the next probability of interest. The comparison of this probability with the above probability is a measure of effectiveness of the training instrumentation system. If the instrumentation system significantly changes the probabilities determined by t and σ_d alone, then the training system does not properly score the trainee, and the system probably provides negative training.

- The final probabilities are the probabilities of a hit and miss when σ_a, the aiming error of the trainee is included. This is the score of the trainee. The best he can do is to match the probabilities computed from t and σ_d alone, that is, he matches the probabilities of a hit and miss (and the kill probability) of the weapon against the target of interest. This probability is not analyzed here, but could be done in a training program.

The next step in this process is to determine the probabilities from the σ measures. Then specific numbers must be applied for each weapon and target combination of interest. Note that it is not necessary to perform this analysis on every target/weapon combination, but to perform it on the most stringent cases that the training instrumentation system must handle. This should only require analysis of a few cases.

Analysis

We initially wish to determine the effectiveness of the particular weapon against the target of interest, given perfect performance by the operator of the weapon system. The parameters outlined above to define this are the target size t, and the dispersion σ_d. We assume the dispersion to be normally distributed. The probability of a hit is then defined by the normal probability integral

$$P_h = \frac{1}{\sqrt{2\pi}}\int_{-\frac{t}{2\sigma_d}}^{\frac{t}{2\sigma_d}} e^{-x^2/2}dx$$

This integral is the area under the normal probability curve (figure B-2) between the limits + and - $t/2\sigma_d$. The density function has been normalized to unit variance.

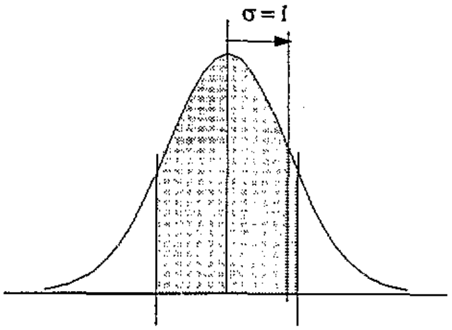

Figure B-2. Normal Probability Integral

When the instrumentation error σ_i is added, the dispersion σ_d increases to

$$\sigma_{di} = \sqrt{\sigma_d^2 + \sigma_i^2}$$

With this increase in overall dispersion due to instrumentation, the hit probability decreases to a value P_{hi}. These probabilities have been calculated as a function of dispersion and instrumentation error, all normalized to target size. As an example, if the instrumentation error is 0.553 of the target size, $\sigma_i / t = 0.553$. Next, supposing that the ratio of dispersion to target size is 0.4, $\sigma_d / t = 0.4$. The combination of dispersion and instrumentation error then yields $(\sigma_d^2 + \sigma_i^2)^{1/2} / t = 0.6825$. The term $t/(2(\sigma_d^2 + \sigma_i^2)^{1/2})$, which is the upper and lower integration limit of the Gaussian distribution is then equal to 0.733. Without any instrumentation, the upper and lower integration limit is $t/2\sigma_d = 1.25$ (calculated from $\sigma_d / t = 0.4$). From tables of the normal probability integral, the probability of a hit without the instrumentation is then $P_h = 0.789$ and the probability of a hit with the instrumentation is $P_{hi} = 0.536$. The probability of a hit is then reduced by a factor of $P_{hi}/P_h = 0.536/0.789 = 0.68$ by the instrumentation. We define negative training to occur when this probability ratio goes below some pre-determined threshold value.

Using this approach, the instrumentation error normalized to target size (σ_i/t) is plotted versus the dispersion normalized to target size (σ_d/t) in figure B-3, for the defined values of the probability ratio P_{hi}/P_h. At low values of σ_d/t, for which $P_h = 1$, which represents the case of weapons which have high accuracy relative to target size, the probability ratio P_{hi}/P_h is the same as would be predicted for hit probability given only the instrumentation error and target size. Thus, for $\sigma_i/t = 0.5$, the hit probability is 0.68. For inaccurate weapons, represented by large values of σ_d/t, the 1-sigma instrumentation error can grow to larger values.

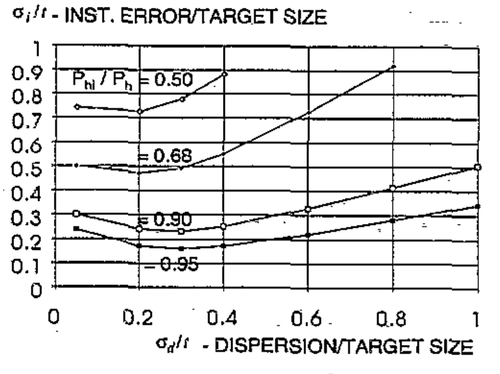

Figure B-3. Instrumentation Error vs. P_{hi}/P_h

This plot also shows that the approach of making instrumentation error equal to dispersion is consistent with this approach, providing dispersion is not less than about 1/3 the target size, and P_{hi}/P_h is between 0.68 and 0.9. Making instrumentation error equal to dispersion is equivalent to a value of P_{hi}/P_h equal to about 0.7 or 0.75.

Figure B-3 allows some simple "rules-of-thumb" to be derived for the relationship of instrumentation error to target size and weapon dispersion. For low dispersion weapons, and a high probability of positive training given by $P_{hi}/P_h = 0.95$, the one-sigma instrumentation error must be about 1/5 of the target size. It can grow to about 1/3 the target size when the dispersion is equal to target size. Following these guidelines assures that the instrumentation system will degrade the hit probability to only 95%, giving a high degree of positive training.

For $P_{hi}/P_h = 0.68$, and low dispersion weapons, the one-sigma instrumentation error can be made equal to half the target size. This is a compromise solution, providing less than ideal training in scoring, but defines the degradation in training to a prescribed level.

A few specific cases are discussed below. A value of $P_{hi}/P_h = 0.68$ is used.

Air-to-surface guns - For a surface target such as a tank or armored personnel carrier, the target size is about 12 by 26 ft (size of a typical tank) . Guns typically have a "cone of fire" which is designed into the gun to ease the aiming problem. This "cone" is distinguished from dispersion because it increases the probability of a hit on the target when successive rounds are fired. A typical angular width for this cone is 4 milliradians. At a range of 1500 ft, this cone subtends about 6 ft. It thereby increases the effective target size somewhat to approximately 20 by 35 ft (adding about 3/4 of this cone size on each side of the target). We therefore approximate the target size as about 20 by 35 ft.

The actual dispersion, as defined, is very low, so the 1-sigma instrumentation error must be about 1/2 the target size, or from 10 to 18 ft (1-sigma) at 1500 ft range. Attitude error in instrumenting the shooter must be 10 to 18/1500 or 6.7 to 12 milliradians 1-sigma (0.4 to .7 degrees 1-sigma). These allowable errors are actually on the high side of what would be ideal, since the probability of a hit was reduced to 68%, and the target size has been "stretched" to the limit. Using $P_{hi}/P_h = 0.95$ would require much better instrumentation, since then the one-sigma instrumentation error would need to be about 1/5 of the target size or 3.5 to 7 ft (1-sigma).

Air-to-air guns - For air-to-air guns, a very similar situation exists. Target size for a tail view of a fighter (a typical shooter/target pairing orientation) is about 9 ft by 43 ft (aircraft height by wingspan). Assuming the same 4 mil firing cone, at 1200 ft range, the firing cone subtends about 5 ft. The effective target size might be increased by the firing cone to about 16 by 50 ft. This yields a positional 1-sigma error of 8 to 25 ft (for the half-target size criteria). Attitude accuracy required also follows the preceding example, requiring about 8 to 25/1200 or 6.7 x 21 mils or .4 x 1.2 degrees 1-sigma error. Again, these errors are on the high side of what would ideally be allowed.

Conclusions

The basic premise of this analysis is that the instrumentation system should give essentially the same probability of a hit as the real weapon would against the real target. We have allowed the hit probability to deteriorate to 0.68 (or to any desired fraction) of what it would in the real situation (0.68 is a compromise - ideally 0.90 or 0.95 would be used). This is the definition of positive training in weapon scoring proposed here. If the probability of a hit deteriorates to much worse than this as compared to the real situation, the training is more of a random numbers game, and it is doubtful that the trainee will benefit much from the experience, at least in weapon proficiency.