

## **Toward Dimensional Analysis Conceptual Modeling for Reusable Modeling Primitive Specification**

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### **ABSTRACT**

Emergent technologies and threats prompt for more robust, nimble, adaptable, and stakeholder-friendly modeling and simulation (M&S) systems-engineering support of simulation-based training and experimentation in the US military. Arbitrary architectural paradigms, inconsistent interoperability protocols and database formats, and proprietary restrictions, among other reasons have given way to models and simulation systems incapable of being reused or used across multidisciplinary domains or communities of interest—severely hampering their return on investment. To be responsive and serve as a unifying and widely-accepted transdisciplinary discipline, the M&S community must provide coherent M&S know-how, leadership, and guidance grounded in tried-and-true science and engineering formalisms that garner multidisciplinary acceptance. We conducted a feasibility study to explore the specification of Reusable Modeling Primitives (RMPs) building on Dimensional Analysis (DA), Design Structure Matrix (DSM) for Complexity Management, and Bond/Causal Graph formal methods. The RMP paradigm underlies a Dimensional Analysis Conceptual Modeling (DACM) framework, conducive to objective specification of model elements and interdependencies. It conduces to methodical reverse engineering, restructure, and reengineering (RE<sup>3</sup>) processes to facilitate, respectively, harvesting codified simuland referents from legacy models, packaging referent information into configurable DA-based primitives amenable to objective fidelity specifications, and conceptual modeling for alternative intended uses leveraging RMPs. DACM facilitates the contextual decomposition of problem spaces mapped to corresponding solution spaces captured in “finger print” DSM matrixes that facilitate the enumeration of required codified simuland referent components—which facilitate validation of problem spaces and of corresponding fidelity requirement specifications underlying M&S solutions. DSM specification matrixes can help reveal not only problem space knowledge gaps but also which knowledge gaps may be resolved with simulation-based analytics and which require further empirical measures—enabling program managers to target their limited research resources. We present the progression and non-trivial realities of model reuse from a demonstration exemplar involving conversion of a rudimentary ship legacy model to a corresponding torpedo alternative model.

### **ABOUT THE AUTHORS**

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**Dr. Eric Coatanéa** is a full tenured professor in the Faculty of Engineering Sciences at Tampere University of Technology. He received a double doctorate in engineering design and product development from Aalto University in Finland and the University of West Brittany in France. His research interests include system engineering, design methodologies, and manufacturing. More specifically, in his research he develops modeling, simulation, optimization and decision support methods for the early design stages using partial and ill-defined knowledge. Currently he integrates Dimensional Analysis and graphs to create novel developments integrating causal graphs, qualitative simulations, Bayesian Networks and Artificial Neural Networks.

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### **INTRODUCTION**

Reuse of computable models and simulation systems has been an elusive challenge that remains a high priority in the U.S. Department of Defense (DoD) modeling & simulation (M&S) community (Davis & Anderson, 2004; Graham, Nelson, & Shea, 2009). To this end, the DoD Modeling & Simulation Coordination Office (M&SCO) and former Joint Assessment and Enabling Capability (JAEC) Office co-sponsored a feasibility study during FY14-16 that included the elaboration of a conceptual modeling framework oriented toward model reuse.

### **Computable Model Reuse Requires Focused Contextual Engineering**

Reuse encompasses a broad range of meaning among M&S stakeholders, thus the feasibility of a universal one-size-fits-all solution is unlikely—although conceptual modeling has been found to promote reuse in the design of M&S applications for certain communities of interest (COIs) [5]. Effective model and simulation system reuse policy that caters to the specific needs of disciplines and military COIs must be deliberate and balanced; that is, reuse of these assets should start with a perspective centered on Service personnel (e.g., soldier-centric) and grounded in sound mathematics, science, and engineering formalisms that generate relevant, high-quality, well-engineered, valid, reusable primitives as building blocks—rather than with wrappers, interoperability, or user-interface panaceas that operate externally or peripherally to models and thus are unlikely to effectively interpret specific functional content.

### **Toward an Objective Computable Model Reuse Paradigm**

The M&SCO/JAEC-sponsored feasibility study produced an initial proof-of-concept version of the Dimensional Analysis Conceptual Modeling (DACM) paradigm, intended to improve objective specifications of Reusable Modeling Primitives (RMPs) with phenomena content and functional interdependencies embedded in the approach. The DACM framework leverages formal methods to include dimensional analysis (DA) to enumerate phenomena content from a problem space, Bond/Causal Graph (BCG) to identify and configure functional interdependencies, and design structure matrix (DSM) to manage the dimensional and functional complexity into a compact “finger print” depiction that facilitates fidelity specifications and model validation. Further, DSM matrix specifications can elucidate functional contradictions and reveal not only problem space knowledge gaps but also which knowledge gaps may be resolved with simulation-based analytics or use of logical and heuristic principles, and which gaps require further experimental measures—enabling program managers to target limited research resources.

### **Objective Conceptual Modeling is Foundational to Reuse**

A foundational premise of the DACM paradigm is that M&S systems engineering that promotes model reuse should, at a minimum, include a conceptual modeling process which deliberately identifies and enumerates the relevant dimensions of the contextual problem space, describes the interdependencies of the various phenomena acting on that problem space, and facilitates the phenomena fidelity and interdependency specifications reflecting the relevant distillation of the problem space and corresponding to the scope (i.e., specific intended uses) of the models underlying a proposed simulation system solution. Arguably, computable models and simulation systems are information-centric such that their information-specific intended uses (I-SIUs) refer to the specific information to be availed (e.g., textual, audio, visual, etc.) for resolving specific decisions to be made, specific questions to be answered, and/or specific cognitive/motor skill to be developed. Further, objective conceptual modeling enables meaningful validation of computable models (i.e., that the “right” computable models are built) when the “right” I-SIUs are elicited, the “right” problem space dimensions are identified, and the right fidelity requirements are specified in an objective manner.

## **Reuse with Methodical Reverse Engineering, Restructuring, and Re-engineering**

The DACM framework is envisioned to enable methodical reverse engineering, restructuring, and re-engineering (RE<sup>3</sup>) processes which facilitate, respectively, harvesting codified simuland referents from legacy models, packaging referent information into configurable DA-based primitives amenable to objective fidelity specifications, and conceptual modeling for alternative intended uses leveraging existing DA-based building-block RMPs. This paper presents an overview of the DACM framework and includes a rudimentary exemplar involving conversion of a ship/water interaction as legacy model to a corresponding torpedo/water interaction alternative model to demonstrate the DACM mechanisms and non-trivial realities of RE<sup>3</sup> processes and model reuse.

### **DIMENSIONAL ANALYSIS CONCEPTUAL MODELING FRAMEWORK**

The DACM framework was conceived for the specification and validation of Reusable Modeling Primitives in M&S engineering processes (Coatanea, 2005; Roca, 2010, 2013). It integrates complementary methodologies related to engineering design, modeling, and simulation. It can be an operative approach to the creation of surrogate models of physical phenomena which can be extended to the training domain, particularly as it applies to adaptable reasoning and critical thinking. The modeling begins with a designation of a system boundary and specification of model intended uses (i.e., what information is needed to support decisions) underlying functional composition. Functional representation is used to represent the sequence of phenomena interactions taking place in the system of interest each of which contributes to the system's overall behavior. In a second stage, the DACM framework transforms an initial functional model into a formalized functional model developed around a limited set of fundamental functional building blocks (Hirtz, Stone, Macadams, Szykman, & Wood, 2001). Third, a causal graph is generated from the formal functional model using causal rules extracted from Bond Graph (Paynter, 1961; Taehyun, 2002) and other research of the authors. Fourth, DA is applied together with a specifically developed algorithm to derive the behavioral equations corresponding to the graph structure. The model can be presented in DSM format and used for qualitative or quantitative simulations, search for knowledge gaps, and detection of functional contradictions. The conceptual modeling process facilitates the specification of a computable model for the system of interest at required measures of quality based on fidelity specifications imposed on the phenomena dimensions identified.

### **Conceptual Modeling**

Conceptual Modeling for models and simulation systems has been recognized as an integral activity within a modeling & simulation (M&S) engineering life-cycle process (Arthur & Nance, 2007; O. Balci & Ormsby, 2007; O. A. Balci, J.D. & Nance, 2008; Robinson, 2004, 2006, 2007). Most M&S practitioners gravitate toward the notion that a conceptual model (CM) somehow relates the context (i.e., content and interdependencies) of a problem space to the scope (i.e., specifications) of a corresponding simulation environment solution space (i.e., computable model and/or simulation system) from which useful synthetic information may be derived.

The Systems Engineering Handbook (INCOSE, 2018) defines concept definition as “the set of systems engineering (SE) activities in which the problem space and the needs and requirements of the business or enterprise and stakeholders are closely examined.” And prominent M&S researchers assert that a CM consists of “high level conceptual constructs ... intended to assist in the design of any type of large-scale complex M&S application” (O. A. Balci, J.D. & Nance, 2008); M&S conceptual modeling generalizes from system architecting depicting the capabilities of a system relative to the operational environment with which it is envisioned to interact (Rechlin, 1991; Robinson, 2004); “the conceptual model is a non-software specific description of the [computable] model (that will be, is or has been developed), describing the objectives, inputs, outputs, content, assumptions and simplifications of the [computable] model” (Kotiadis & Robinson, 2008); “The process of developing a simulation includes understanding what ‘reality’ needs to be simulated, choosing a referent, and deciding how and how much of the referent will be referenced in the simulation” (Gross, Pace, Harmon, & Tucker, 1999).

A CM guides the design and development of a computable model as it enables 1) Assumptions and determination of the components that will be included in a computable model, 2) The approach to modeling system or phenomenon behavior, 3) The elimination of unimportant features, and 4) The selection of interface and boundary types. Most importantly, conceptual modeling identifies the components in the reality of interest which are initially anticipated to have significant effects on the system's response (ASME, 2006).

A CM is not an executable or computable model such that it does not compute or generate synthetic information. It does, however, inform (and may be informed by) other activities within an M&S engineering life-cycle process such as the decomposition of the problem space, the elaboration of intended uses, derivation of functional requirement specifications, and the early conceptualization of a solution space (i.e., high-level content and architecture of a simulation system). Figure 1 distinguishes a CM from a computable model along with intermediate modeling activities. In contrast to a CM, a computable or executable model (of some natural or man-made system or phenomenon) computes or generates useful synthetic information toward satisfying a set of intended uses. A computable model "... is a software specific design and software representation of the conceptual model ... [it] is not strictly part of conceptual modelling, but it does ... embody the conceptual model within the code of the [computable model]" Robinson [16]. It is "the numerical implementation of the mathematical model that will be solved on a computer to yield the computational predictions (simulation results) of the system response" (ASME, 2006).

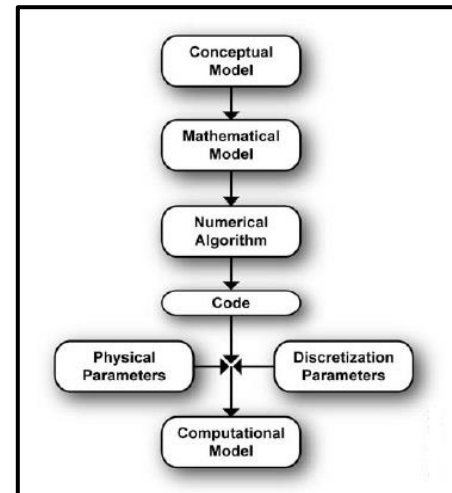
As an architecting artifact, a CM serves as a repository for stakeholders to iteratively capture the context (i.e., relevant and significant aspects or dimensions) of a problem space as well as a schema of the scope of capabilities of an envisioned M&S solution space (i.e., computable model and/or simulation system) and the intended uses of its acquirer. Detailed M&S functional requirements for a computable model may then be derived from a CM to shape the required functionalities of the M&S solution space and provide direction to the design and development team. The Distributed Simulation Engineering and Execution Process (DSEEP) (IEEE, 2011), a generalized systems-engineering standard to distributed simulation, indicates that "The conceptual model provides an implementation-independent representation that serves as a vehicle for transforming objectives [i.e., intended uses] into functional and behavioral descriptions [i.e., requirements for system and software designers]."

M&S requirements derived from a CM direct the design and development of computable models which amount to encoded selections of a simuland's referent (i.e., body of knowledge about the thing being simulated) that generate computed synthetic information toward resolving intended uses. Requirement specifications for computable models are centered on fidelity specifications which enumerate the dimensions or entities involved in the simuland as well as their units of measure, accuracy (i.e., closeness to truth), precision (i.e., variance of accuracy), resolution (i.e., smallest detectable changes of measure), tolerance, error, sensitivity, aggregation, granularity, range of operation, and boundary conditions among other measures of quality (Harmon & Youngblood, 2005; JCGM, 2008). Fidelity refers to "... the degree to which [a computable model] reproduces a referent ... provides the best means to specify a particular set of requirements ... [and] is a critical enabler for the practical reuse of simulations." (Gross et al., 1999).

### Dimensional Analysis

DA is a branch of algebraic theory with a broad range of applications in the physical, life, and social sciences (Stahl, 1961). It was formalized over a hundred years ago by Lord Rayleigh (1915) and Edgar Buckingham (1914) but was employed in earlier forms by the likes of Newton (1686), Fourier (1822), Maxwell (1865), and many others. DA may be arguably one of the most foundational modeling methods yet it is foreign to most M&S practitioners, and most of those familiar with the term consider it simply a way to convert systems of units (e.g., meters to yards) or for ensuring the homogeneity of equations (i.e., adding apples to apples and not apples to oranges) (Stahl, 1961).

DA is very powerful and well suited for M&S systems engineers as it helps to reduce the number and complexity of experimental variables involving physical phenomena, while requiring modest mathematical knowledge. It is founded on basic principles of science and establishes the functional relationship in the data from defined dimensions; in contrast to something like regression analysis which requires that a functional relationship be defined up front (linear or nonlinear exponential, trigonometric, polynomial, etc.) and from which an unknown relationship cannot be produced from known variables (Bruno, Doctorow, & Kappner, 1981).



**Figure 1. ASME Model Development Progression (ASME, 2006)**

### Dimensions and Units

Measurement is the basis for DA, as is the basis for science and engineering, and it involves dimensions and units. A dimension is the measure which expresses a physical variable qualitatively whereas a unit expresses a physical quantity so as to relate a number to a dimension. The dimension of a physical variable is independent of the units in which it is measured. For example, length is a dimension of distance, height, depth, and so on whereas foot, meter, and mile are units for measuring length.

### Fundamental and Composite Dimensions

The physical world consists of fundamental dimensions such as time, length, temperature, mass, and charge. These fundamental dimensions may be combined to derive composite dimensions such as area, volume, velocity, acceleration, force, density, etc. A dimension is said to be independent if it cannot be expressed by other dimensions. For example, distance, time, and velocity are physical quantities that are not dimensionally independent because any one can be derived from the remaining two. Physical dimensions have been used as analogs to “dimensionalize” life and social science phenomena (Bruno et al., 1981; Stahl, 1961).

### Dimensional Homogeneity

DA is based on the Principle of Dimensional Homogeneity which states that each of the terms of an equation describing a physical phenomenon must have the same dimension (i.e., adding apples to apples and not apples to oranges). For instance, equation (1) describes the linear distance ( $x$ ) of dimension Length (L) traveled by a moving particle over a period ( $t$ ) of dimension Time (T), subject to an initial velocity ( $v_0$ ) of dimension Length per unit Time (L/T or  $LT^{-1}$ ) and acceleration ( $a$ ) of dimension Length per unit Time per unit Time ( $L/T^2$  or  $LT^{-2}$ ).

$$x = v_0 t + \frac{1}{2} a t^2 \quad (1)$$

Each of the terms in equation (1) have dimension of Length (L) as confirmed by the following expressions:

$$[x] = L \quad (2)$$

$$[v_0 t] = LT^{-1}T = L \quad (3)$$

$$[\frac{1}{2} a t^2] = LT^{-2}T^2 = L \quad (4)$$

### Dimensionless Variables

A dimensionless variable refers to a bare number or quantity which has no physical dimension such as  $\pi$  (3.14 ...),  $i$  (imaginary number;  $i^2 = -1$ ),  $e$  (Euler’s number, 2.71828), Avogadro’s number ( $6.022 \times 10^{23}$ ), etc. Dimensionless quantities are often used in physics to simplify the characterization of systems with multiple interacting physical phenomena since certain physical constants can be normalized to 1 if appropriate units for fundamental dimensions are chosen. Most people are familiar with the dimensionless number  $\pi$  (3.14 ...) as a proportionality factor which relates the diameter ( $d$ ) and circumference ( $C$ ) of a circle (dimensions of Length (L)) by the relation  $C = d \pi$ . Similarly, other such dimensionless “ $\pi$ ” numbers simplify phenomena relationships, such as the Fresnel number which relates the number of half-period zones in wavefront amplitude for an electromagnetic wave passing through an aperture.

### II Theorem

The fundamental II Theorem of DA states that a physical process described by a dimensionally homogeneous relation involving  $n$  dimensional variables, such as equation (5), has an equivalent relation involving  $(n - k)$  dimensionless variables, such as equation (6), where  $k$  is the maximum number of dimensionally-independent variables and is usually equal to but never greater than  $n$ . The dimensionless  $\pi$  numbers in equation (6) are derived from  $(k + 1)$  combinations of  $x$  variables thereby reducing the number of independent variables by  $k$ .

$$x_1 = f(x_2, x_3, \dots, x_n) \quad (5)$$

$$\pi_1 = F(\pi_2, \pi_3, \dots, \pi_{n-k}) \quad (6)$$

The  $\Pi$  Theorem enables a more efficient approach to identifying, organizing, reducing, and managing the phenomena variables of a problem space from which different informational perspectives (e.g., I-SIUs) can be considered.

For instance, a rudimentary example (Eddey, 1945) considered the practical problem of relating water resistance to ship motion relative to ship velocity. In general, force of water resistance depends on the shape of the hull of the ship, but if a comparative analysis is being made of ships with similar hulls then the hull shape is effectively normalized and need not be considered further such that ship size may be characterized by a linear dimension and the relevant quantities for DA are:

$$[V] = \text{velocity of ship} = LT^{-1} \quad (7)$$

$$[P] = \text{force on the ship} = MLT^{-2} \quad (8)$$

$$[S] = \text{immersed surface area} = L^2 \quad (9)$$

$$[l] = \text{linear length} = L \quad (10)$$

$$[\nu] = \text{kinematic viscosity of water} = L^2T^{-1} \quad (11)$$

$$[r] = \text{density of water} = ML^{-3} \quad (12)$$

These expressions reveal six quantities ( $n = 6$ ) in 3 dimensions ( $k = 3$ ), such that 3 dimensionless  $\pi$  numbers ( $n - k = 3$ ) can be derived. Derivation of  $\pi$  numbers begins with the general  $\pi$  expression using arbitrary exponents for each of the participating quantities (equation 13), where the dimensional equivalents are substituted (equation 14).

$$\pi = P^a \cdot S^b \cdot l^c \cdot \nu^d \cdot V^e \cdot r^f \quad (13)$$

$$\pi = (M^a L^a / T^{2a}) (L^{2b}) (L^c) (L^{2d} / T^d) (L^e / T^e) (M^f / L^{3f}) \quad (14)$$

The  $\pi$  number must be dimensionless such that the exponents of each of the participating dimensions (M, L, and T) must add to zero as follows:

$$a + f = 0 \quad (15)$$

$$a + 2b + c + 2d + e - 3f = 0 \quad (16)$$

$$-2a - d - e = 0 \quad (17)$$

Since there are six variables in three equations, the values of three variables (referred to as the repeating variables) will need to be assigned arbitrarily (this is the tricky part) and the remaining three variables resolved by substitution in equations 15-16. For example, the first  $\pi$  number may be derived by assigning  $a = 1$ ,  $c = 0$ , and  $e = 0$  arbitrarily, resulting in  $b = 0$ ,  $d = -2$ , and  $f = -1$ , and substituting the values into equation 13:

$$\pi_1 = P / (V^2 \cdot r) \quad (18)$$

Similarly, the second  $\pi$  number may be derived by assigning  $a = 0$ ,  $b = 0$ , and  $e = 1$  arbitrarily, resulting in  $c = 1$ ,  $d = -1$ , and  $f = 0$ ; the third  $\pi$  number may be derived by assigning  $a = 0$ ,  $b = 1$ , and  $e = 0$  arbitrarily, resulting in  $c = -2$ ,  $d = 0$ , and  $f = 0$ ; and the equivalent relation (equation 21) is complete. This derivation of repeating variable values was automated as part of the DACM feasibility study (Coatanea, Roca, Mokhtarian, Mokammel, & Ikkala, 2016).

$$\pi_2 = lV/\nu \quad (19)$$

$$\pi_3 = S/l^2 \quad (20)$$

$$F(\pi_1, \pi_2, \pi_3) = F\left(\frac{P}{V^2 \cdot r}, lV/\nu, S/l^2\right) = 0 \quad (21)$$

If further comparative analysis is conducted reducing on ships with similar displacement of water (i.e., similar length ( $l$ ) and immersed surface area ( $S$ )) then the third  $\pi$  number ( $\pi_3$ ) is normalized for all ships and may be disregarded in this example, such that the equivalent relation becomes:

$$F\left(\frac{P}{V^2 \cdot r}, lV/\nu\right) = 0 \quad (22)$$

$$\text{i.e.,} \quad \frac{P}{V^2 \cdot r} = F(lV/\nu) \quad (23)$$

This reflects the simplicity afforded by the  $\Pi$  Theorem in that a single curve (relating  $\frac{P}{V^2 r}$  and  $\frac{lV}{v}$ ) can show the effects and significance of five variables. In contrast, a more conventional approach plotting  $P$  vs.  $V$  with  $l, r,$  and  $v$  as parameters, with a mere five values for each parameter, would require  $5^3 = 125$  curves. The DACM framework is based on the premise that these  $\pi$  numbers may serve as building blocks to support the specification of RMPs (Reusable Modeling Primitives) and conceptual modeling for models and simulation systems.

### Bond/Causal Graphs

Bond graphs are domain-independent graphical descriptions of dynamic behavior, based on energy and energy exchange, used to describe physical systems from different domains (e.g., electrical, mechanical, material) in a consistent manner such that analogies of physical concepts enable analogies between domains. Bond graph models have been around for nearly 60 years and have evolved to a systems theory since they are not only very versatile and instrumental for modeling complex engineering systems that usually involve multiple domains but also reusable as they are a form of object-oriented modeling (Broenink, 1999). Bond graphs are directed and labeled graphs, amenable to labeled transition systems, with vertices representing idealized physical phenomena and edges (i.e., bonds) representing idealized energy connections between components. Bonds are bi-directional signal connections with a power direction and a computational causality direction. Bond graph analogies across domains are possible from the recognition that various physical systems can be represented by similar sets of differential equations (Broenink, 1999).

The underlying assumptions are the conservation law of energy and the premise that it is possible to separate system properties and denote them distinctly, implying that the model of a concrete part is not necessary only one concept but rather potentially a set of interconnected concepts (Kron, 1963; Paynter, 1961). Constraints on the connected bonds depend on the kind of equations describing the elements and a systematic procedure for causal analysis of bond graphs is necessary to determine the signal direction of the bonds resulting in a “causal bond graph” (Broenink, 1999). The causal ordering procedure can be automated.

Figure 2 illustrates the modeling sequence for a rudimentary mechanical system. The phenomena involved in the system concept includes voltage and current input to the electric motor, converted to electromechanical power, converted to torque and angular velocity applied to the pump shaft, converted to fluid power which drives the pump, exerting pressure on the fluid, resulting in fluid output flow and pressure. These phenomena are isolated and enumerated in the power flow diagram which enables a more-detailed consideration of the phenomena involved, such as inertia from pump moving parts, capacitance from fluid compressibility, resistance from friction among moving parts, and so on resulting in the final causal bond graph representation (Samantaray, 2001). The DACM framework extends the BCG to form a DACM causal graph. It is important to recognize that a causal graph constitutes fundamental transformations and connecting units which form a set of elementary building blocks.

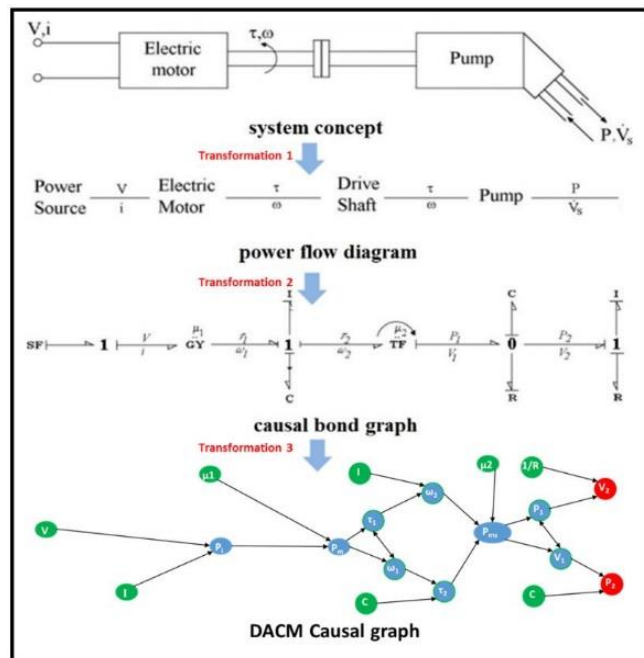
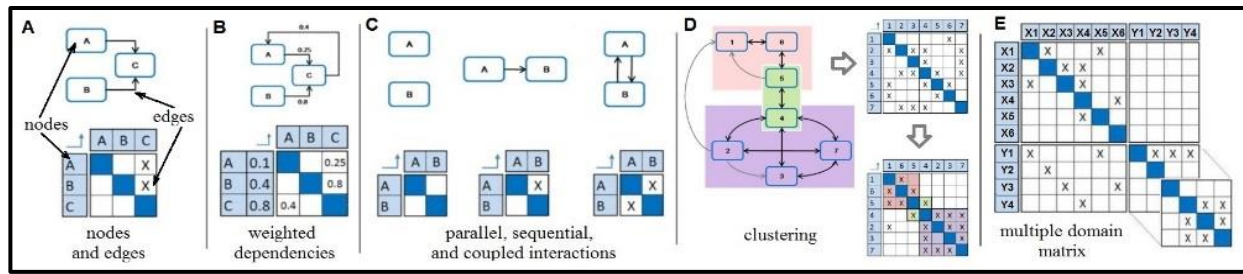


Figure 2. BCG Example (Samantaray, 2001)

### Design Structure Matrix

The DSM is a tool for analysis and management of complex systems by enabling the identification, enumeration, and classification of entities constituting a system and accounting for the interdependencies among them. The basic DSM is a square matrix amenable to powerful analyses involving partitioning and clustering (e.g., to facilitate modularity, aggregation, and composability), various types of directed graphs (digraphs), and systems of equations among others. Digraphs in particular are commonly used in depicting and managing complex structures, but a major advantage of DSM representations is a compact but extensible format —particularly relevant when managing large systems— that

enables systematic mapping, manipulation, isolation, and analysis of system entities using matrix algebra and other simple mathematics. The basic DSM may be extended from a matrix that relates entities of the same kind (e.g., functions to functions), to a Domain Mapping Matrix (DMM) which enables mappings between two different views of a system (e.g., functions to components), and to a Multiple Domain Matrix (MDM) which combines the DSM and DMM to afford a complete system model enabling system analysis across multiple domains. Figure 3 depicts some of the possible DSM properties (Lindemann, 2016). These properties not only capture the dimensional content and functional interactions of DA and BCG representations but also, when implemented in the DACM framework, visualize “finger print” model specifications that can be correlated to I-SIUs and that can reveal functional contradictions and space knowledge gaps to help program managers determine when simulation-based and/or field experimentation is appropriate.



**Figure 3. Some DSM Properties (Lindemann, 2016)**

## DACM FRAMEWORK OVERVIEW

At the heart of the computable-model and simulation-system reuse challenge, and of the DACM proposition to resolve it, is the formulation of a conceptual modeling strategy that enables the codifying and modularizing of simuland referent information in a manner that is versatile and composable. And, as presented previously, a significant aspect of conceptual modeling for computable models and simulation systems includes understanding what ‘reality’ needs to be simulated, choosing a referent, and deciding how and how much of the referent will be referenced in the simulation. To this end, the DACM framework offers a progression for identifying the relevant content and interdependencies of a problem space relative to a set of I-SIUs and for engineering composable building blocks (i.e., RMPs) amenable to objective fidelity specifications.

Composability researchers have dismissed the illusion of pure plug-and-play paradigms as unrealistic propositions; calling instead for more robust systems-engineering processes that address computable-model and simulation-system complexity to minimize the composition time and level of effort leading to more realistic composable components (Davis & Anderson, 2004). And it has been determined that the greatest leverage in system architecting is at module interfaces (Rechtin, 1991) leading to increased acceptance of high-cohesion (inside a component) and low-coupling (among components) design strategies.

Supporting the DACM paradigm on the one hand, the DSM method is instrumental in identifying and managing high cohesion components by way of its “clustering” feature (Fig 3-D). Clustering relocates elements within a DSM matrix toward consolidating DSM interactions as clusters of internal functionalities and minimizing or eliminating interactions among separate clusters (Fernandez, 1998; Sharman and Yassine, 2003; Yu et al., 2003). DSM clusters effectively serve to configure computable-model modules from which further refinement and specification of RMPs can be realized.

On the other hand, DA offers another type of “clustering” by way of the dimensionless  $\pi$  numbers explained previously. These  $\pi$  number “clusters” amount to composites of consolidated variables that significantly reduce the number of variables to be considered and managed in analysis and experimental design. This reduction translates to substantial benefits that include methodical discovery of relevant and superfluous variables, identification of functional contradictions, more-effective conceptual modeling based on information-based intended uses, objective derivation of relevant fidelity specifications, optimal modular design based on identification of functional interdependencies and coupling, reduced experimental test iterations, targeted validation of fidelity specifications, and design for functional-verification testability among others. Further, the transparency afforded by DA and DSM

clustering enables not only the identification of knowledge gaps but also the differentiation between knowledge gaps resolvable through simulation-based analytics and those requiring empirical measures. It is also worth noting that the DACM framework may be adapted into an RE<sup>3</sup> process to repurpose or reconstitute legacy computable models and simulation systems from original to alternative intended uses as well as provide a basis for more robust rapid prototyping and agile development.

The key transformation steps are outlined here and summarized in the following sections using rudimentary exemplars (rough correspondence illustrated in Figure 4):

1. Identify problem space and intended uses
2. Conduct DA to enumerate relevant variables and dimensions
2. Derive causal graph from variables and functional representation
3. Convert causal graph of variables to variable-interdependency DSM matrix
4. Employ DSM and propagation algorithm to identify coupling and contradictions
5. Derive DMM matrix to relate variables and dimensions
6. Employ DA and DSM to derive  $\pi$  numbers (clusters) from Variable-Dimension DMM
7. Create  $\Pi$  DSM matrix from derived  $\pi$  numbers
8. Employ DSM clustering to analyze different  $\pi$ -to- $\pi$  relationships
9. Illustrate knowledge gaps through DSM fingerprint specifications
10. Compare DSM fingerprint specifications of legacy and alternative models to indicate reuse profiles

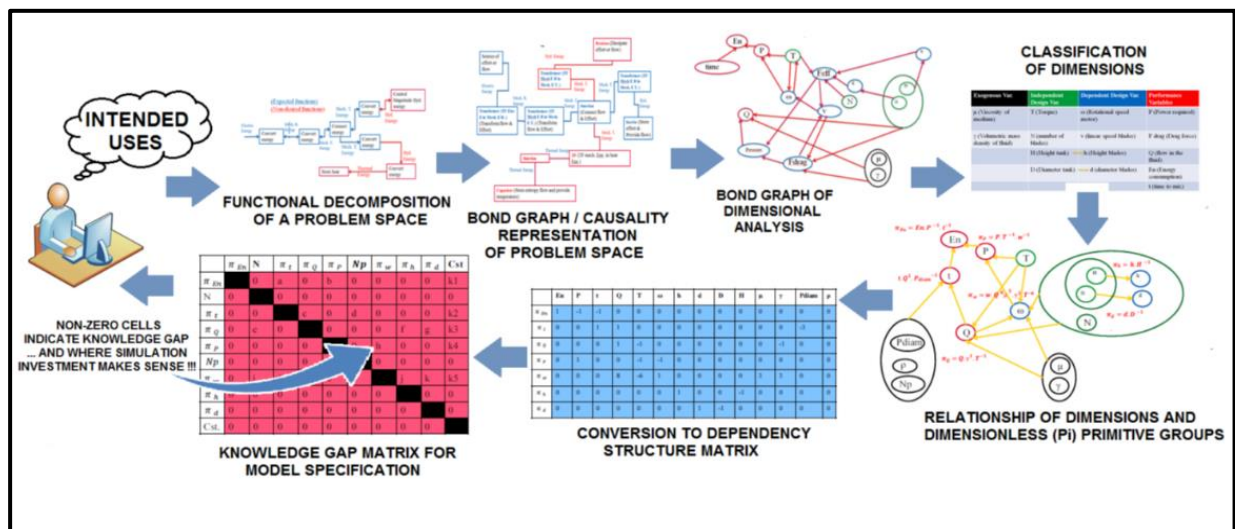


Figure 4. Notional Progression of DACM Framework

**Steps 1-2**

To illustrate, we use the sample problem space relating water resistance to ship motion relative to ship velocity (Edey, 1945) as was described previously in the Dimensional Analysis section. The intended uses here require a computable-model of the interaction of the ship with the seawater to inform the ship design’s aim of maximizing the ship velocity (V) while concurrently minimizing the drag force (F).

The DA has been conducted previously and captured in equations 7-12, involving 6 variables (l, S, v, V, r and F) and 3 dimensions (i.e. M, L, and T), implying  $N = 6-3 = 3$  dimensionless groups or  $\pi$  numbers.

**Steps 3 - 4**

From the DA enumeration of relevant variables and dimensions from the previous steps, a functional model (Figure 5-A) is derived that identifies the desired and non-desired functions resulting from the interaction of the ship with its seawater environment –and further identifies state variables (i.e., displacement, momentum, material and geometric

properties) and power variables (i.e., effort and flow). The resulting set of variables is categorized as independent design variables (green), performance variables (red), and exogeneous variables imposed by the environment on the system (black). A causal graph (Figure 5-B) is then derived from the functional model and converted to a more compact and versatile DSM format (Figure 5-C) amenable to linear algebra and matrix transformations.

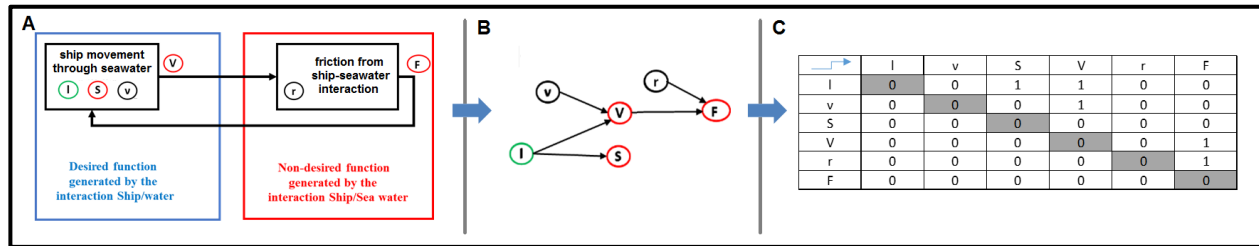


Figure 5. A-Functional Model, B-Causal Graph, and C-DSM of Legacy Ship Model

Steps 5-9

Coupling and contradictions are identified by DSM clustering and by propagating variable objectives (e.g., minimizing, maximizing, etc.) using previous methods (Bhaskar & Nigam, 1990) in the context of causal graphs and DSM (Coatanea et al., 2016). In this trivial case study, the drag force (F) is minimized, the velocity (V) maximized, and no specific objective assigned to the immersed surface area S. Figure 6A shows the propagation of three contradictory variable objectives (i.e., I, v and V; shaded in yellow) involving positive and negative coupling corresponding to equation 19. Contradiction detection provides useful information related to an optimization process in an M&S context. An optimization process is required only for variables exhibiting contradictions while the other variables can be fixed to their minimal or maximal values.

The traditional process for deriving  $\pi$  numbers presented in the Dimensional Analysis section (equations 13-20) is automated in DACM using DMM manipulations and linear algebra. Figure 6 shows the trivial progression from causal graph to DMM relating variables and dimensions, to the composition of DSM relating dependencies among  $\pi$  numbers corresponding to equations 18-20.

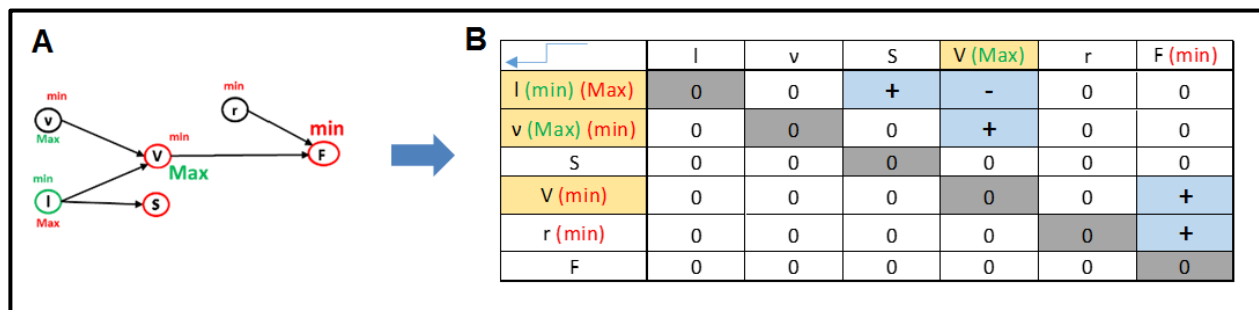


Figure 2. A-Propagation of Variable Objectives in the Causal Graph, and B-Identification of Variable Coupling (shaded blue) and Variable Contradictions (shaded yellow) in DSM

The significant reduction of analytical and experimental iterations can be seen at this juncture even with this trivial case study. That is, a reasonable analysis of this sample problem space would specify that the ship velocity (V) is a function of the five variables in equations 8-12 as depicted in equation 24. If analysis or experimentation was conducted on this basis, then a variation of only P would provide a single curve relating V and P. Repeating this approach with each of the remaining variables and assigning a minimum of five points to plot a curve for each combination of variables would require  $5^5 = 3125$  analytical/experimental points or 6250 readings (for 2 coordinate axis plots). In contrast, analysis based on the three derived  $\pi$  numbers would reveal that only two of them ( $\pi_2$  and  $\pi_3$ ) relate ship velocity (V) to drag force (F) thereby requiring only one plot of five points and ten total readings – a substantial reduction in analytical/experimental effort (Gibbins, 2011).

$$V = f(P, S, l, v, r) \tag{24}$$

Identification of specific knowledge gaps is important to help guide analytical and experimental resources. The work conducted in the DACM feasibility study produced DSM-based in tandem with other analytical methods to help identify knowledge gaps and present them conveniently in DSM format as non-zero cells (depicted in the conceptual pink DSM extracted from a separate case study in Figure 7). Knowledge gaps identified by DA typically require field experiments to discover the nature of unknown variable relationships but may be resolvable by simulation-based experimentation. A heuristic has been elaborated to explore the nature of  $\pi$ -number relationships to possibly resolve knowledge gaps of variable relationships analytically (Szyrtes & Rozsa, 2006). For example, an arbitrary relation between  $\pi_1$  and  $\pi_2$  takes the general form  $\pi_1=f(\pi_2)$ , generally written in the form of a power relationship,  $\pi_1=c.\pi_2^\epsilon$ , which in the ship exemplar is  $\frac{S}{l^2} = c \left(\frac{LV}{\gamma}\right)^\epsilon$ . A reasonable assumption would be that a change in the ratio  $S/l^2$  is proportional to the change of  $V$ , making  $\epsilon$  equal to 1. The heuristic/assumption reduces the need for only one experiment to determine  $\epsilon$ ; the relation between  $\pi_2$  and  $\pi_3$  is more complex and requires more experiments to discover the relationship; heuristics can potentially reduce the need for experiments but cannot eliminate them completely.

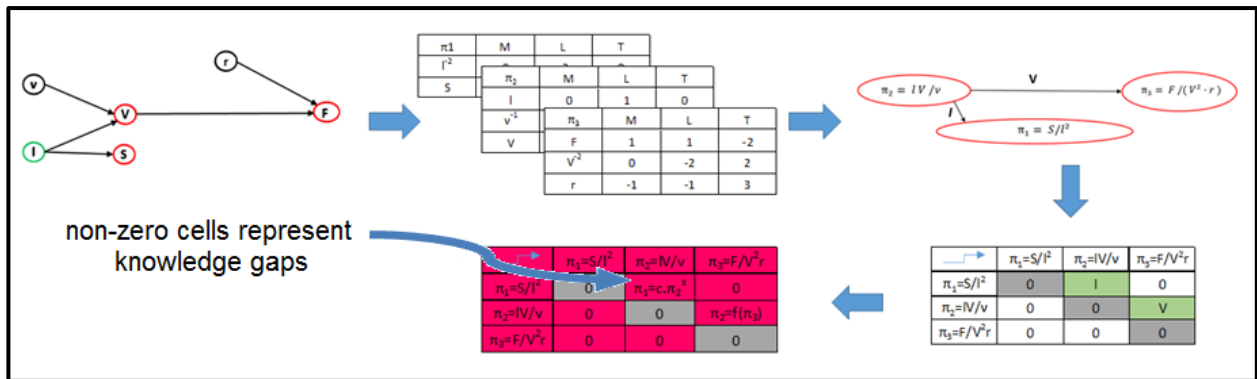


Figure 7. Derivation of  $\pi$  Number Relationships and Knowledge-Gap Finger-Print DSM

Step 10

To illustrate the analysis of and specification for reuse afforded by the DACM framework, we examine the sample case study of the ship interaction with seawater as a “legacy model” and introduce the interaction of a torpedo and the seawater as an “alternative model” with the same intended uses as described in Steps 1-2. Without the availability of the  $\pi$ -number artifacts (RMPs) derived from the legacy model, the torpedo alternative model would follow the same

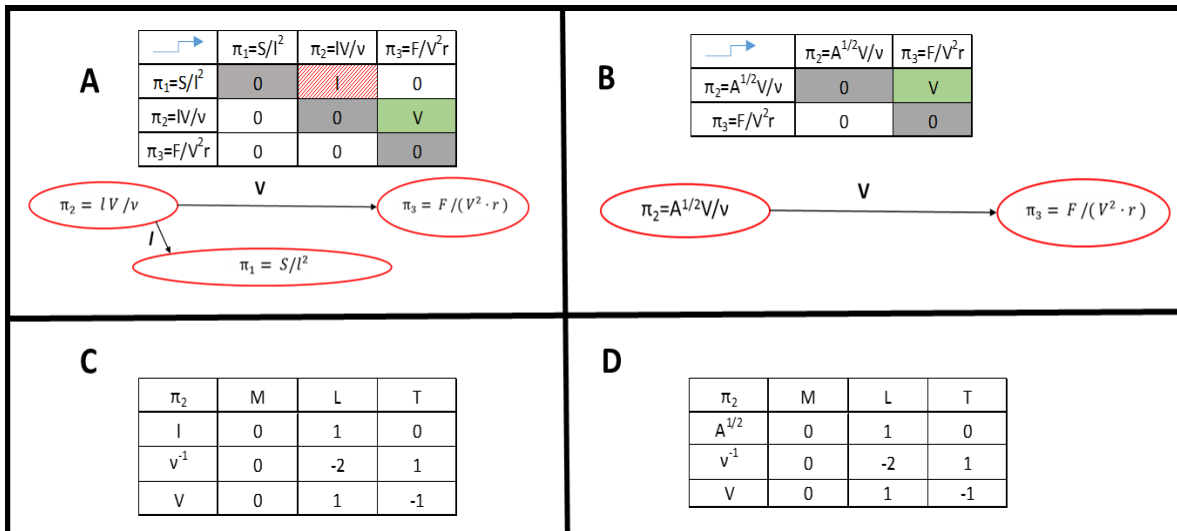


Figure 8. A - Legacy Model  $\pi$  DSM of ship/water interaction, B - Alternative Model  $\pi$  DSM of torpedo/ water interaction, C-D – Representative Variable-Dimension DSMs  $\pi_2$  for Legacy and Alternative Models

analytical progression (Steps 1-9) and Figure 8-A illustrates the resulting DSM relating  $\pi$ -number dependencies. Figure 8-B presents the alternative model counterpart. In Figure 8-A the red shading identifies the legacy model  $\pi$  number that is superfluous to the alternative model, revealing that 2/3 of the legacy model  $\pi$  numbers are common and reusable, such that their derivation is redundant and unnecessary. The DACM framework automates the comparison of DSM matrices using linear algebra (e.g., Figure 8 C-D for the computation of the  $\pi_2$  RMP named; note the similarities (i.e. same form of the equation) and difference (i.e.  $l$  replaced by  $A^{1/2}$ ) between the two primitives.

## DISCUSSION

The notion of reusable computational resources has been around for a long time (e.g., object-oriented programming, application program interfaces, interoperability protocols, database standardization, etc.). Perhaps one of the principal differences between general computer science and M&S is the need for domain expertise in the latter to conduct effective conceptual modeling that captures I-SIUs and conduces to objective fidelity specifications of relevant phenomena and system (natural or man-made) interdependencies—as well as the validation of said specifications.

It is not uncommon in the M&S user (and developer) community to think of a simulator (or rather of the underlying computable models) to be nothing more than a collection of equations with an arbitrary number of variables that can be manipulated endlessly until a solution is reached. Such approach resembles the Infinite Monkey Theorem which states that a monkey typing randomly on a keyboard for an infinite amount of time will eventually type Shakespeare's soliloquy. From that perspective also comes the notion that model reuse should be relatively simple and reduced to a parameter configuration exercise.

The rudimentary examples presented in this paper, used to elucidate not only the mechanisms (i.e., DA, BCG, DSM) underlying the DACM paradigm but also the critical thinking required to resolve phenomenological and functional content, demonstrates that the specification, validation, refurbishment, maintenance, and repurposing of computable models is anything but trivial—and requires a working knowledge of science and engineering principles.

DACM-based M&S systems engineering offers a conceptual-modeling approach in which representation of complex problems with  $\pi$  numbers as RPMs offers substantial benefits to reduce the burden of analytical/experimental efforts. Further, it offers an alternative approach that can facilitate dialog between M&S systems engineering liaisons and subject matter experts based on a foundational but robust science and engineering framework.

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## REFERENCES

- Arthur, J. D., & Nance, R. E. (2007). Investigating the Use of Software Requirements Engineering Techniques in Simulation Modeling. *Journal of Simulation*, 1(3), 159-174.
- ASME. (2006). *Guide for Verification and Validation in Computational Solid Mechanics*.
- Balci, O., & Ormsby, W. F. (2007). Conceptual Modelling for Designing Large-Scale Simulations. *Journal of Simulation*, 1, 175-786.
- Balci, O. A., J.D., & Nance, R. E. (2008). Accomplishing Reuse with a Simulation Conceptual Model. *Proceedings of the 2008 Winter Simulation Conference*, 959-965.
- Bhaskar, R., & Nigam, A. (1990). Qualitative Physics Using Dimensional Analysis. *Artificial Intelligence*, 45, 73-111.
- Broenink, J. F. (1999). Introduction to Physical Systems Modelling with Bond Graphs. Retrieved from
- Bruno, J. E., Doctorow, O., & Kappner, C. H. (1981). Use of Dimensional Analysis in Social Science Research. *Socio-Economic Planning Sciences*, 15, 95-99.
- Coatanea, E. (2005). *Conceptual Modeling of Life Cycle Design: A Modeling and Evaluation Method Based on Analogies and Dimensionless Numbers*. ( PhD Dissertation).
- Coatanea, E., Roca, R. A., Mokhtarian, H., Mokammel, F., & Ikkala, K. (2016). A Conceptual Modeling and Simulation Framework for System Design. *Computing in Science & Engineering*, 18(4), 42-52.

- Davis, P. K., & Anderson, R. H. (2004). *Improving the Composability of Department of Defense Models and Simulations*. Retrieved from Santa Monica, CA:
- Eddey, E. E. (1945). Some Engineering Applications of the Buckingham Pi Theorem. *Ohio State Engineer*, 28(3), 7-8, 22-24, 32.
- Gibbins, J. C. (2011). *Dimensional Analysis*. London: Springer.
- Graham, K. L., Nelson, J. B., & Shea, D. P. (2009). *Business Models to Advance the Reuse of Modeling and Simulation Resources*. Retrieved from Alexandria, VA:
- Gross, D., Pace, D., Harmon, S., & Tucker, W. (1999). Why Fidelity? *Spring 1999 Simulation Interoperability Workshop Paper - 99S-SIW-168*, 1055-1061.
- Harmon, S., & Youngblood, S. (2005). A Proposed Model for Simulation Validation Process Maturity. *Journal of Defense Modeling and Simulation*, 2(4), 179-190.
- Hirtz, J., Stone, R. B., Macadams, D. A., Szykman, S., & Wood, K. L. (2001). A Functional Basis for Engineering Design: Reconciling and Evolving Previous Efforts. *Research in Engineering Design*, 13, 65-82.
- IEEE. (2011). Recommended Practice for Distributed Simulation Engineering and Executive Process (DSEEP) (pp. 1730-2010): Institute of Electrical and Electronics Engineers
- INCOSE. (2018). The International Council on Systems Engineering. Retrieved from <http://www.incose.org/>
- JCGM. (2008). International Vocabulary of Metrology. Retrieved from [http://www.bipm.org/utils/common/documents/jcgm/JCGM\\_200\\_2008.pdf](http://www.bipm.org/utils/common/documents/jcgm/JCGM_200_2008.pdf)
- Kotiadis, K., & Robinson, S. (2008). Conceptual Modelling: Knowledge Acquisition and Model Abstraction. *Proceedings of the 2008 Winter Simulation Conference*, 951-958.
- Kron, G. (1963). *Diakoptics: The Piecewise Solution of Large-Scale Systems*. London: Macdonald & Co.
- Lindemann, U. (2016). The Design Structure Matrix. Retrieved from <http://www.dsmweb.org>
- M&S Priority Objectives. (2012). DoD M&S Steering Committee. Washington, DC.
- Paynter, H. M. (1961). *Analysis and Design of Engineering Systems*. Cambridge, MA: MIT Press.
- Rechtin, E. (1991). *Systems Architecting - Creating & Building Complex Systems*. New Jersey: Prentice Hall.
- Robinson, S. (2004). *Simulation: The Practice of Model Development and Use*. West Sussex, England: John Wiley & Sons, Ltd.
- Robinson, S. (2006). Issues in Conceptual Modeling for Simulation: Setting the Research Agenda. *Proceedings of the 2006 Operational Research Society Simulation Workshop*, 165-174.
- Robinson, S. (2007). The Future's Bright the Future's ... Conceptual Modeling for Simulation! *Journal of Simulation*, 1(3), 149-152.
- Roca, R. (2010). Exploring DSM to Support Systems Engineering of Composable Simulation Environments. *Proceedings of the 12th International DSM Conference*, 419-425.
- Roca, R. (2013). M&S Conceptual Modeling as an Enabler of M&S Reuse, Agile, and Open-Source: A TRS Initiative in Response to the M&S SC Priority Objectives. *M&S Journal - The Reuse Issue*, 20-27.
- Samantaray, A. K. (2001). About Bond Graphs - the System Modeling World. Retrieved from <http://www.bondgraphs.com/about.html>
- Stahl, W. R. (1961). Dimensional Analysis in Mathematical Biology I. General Discussion. *Bulletin of Mathematical Biophysics*, 23(4), 355-376.
- Taehyun, S. (2002). *Introduction to Physical System Modelling Using Bond Graphs*.